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«Long-Term Contracts in Carbon and Electricity Markets with Frictions »

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Abstract

In the context of the urgent need for decarbonization, this paper lays the foundation for investigation the interaction between long-term contracts and carbon pricing in the electricity sector, focusing on their effects on investment dynamics and decarbonization. An hybrid modeling framework is developed with an optimization model and a simulation model. It endogenizes the carbon price to study these interactions under various market conditions (optimization and simulation models). The results indicate that intertemporal flexibility through emissions banking stabilizes carbon prices and supports sustained investment. Market frictions, such as limited foresight, are shown to impede decarbonization and investment, but a complementarity remuneration mechanism for decarbonized assets such as long-term contracts can mitigate these deficits. The findings underscore the importance of well-designed policies and contracts for achieving optimal decarbonization paths. Allowing banking in a quantity-based carbon policy leads to lower system costs even in presence of market frictions such as limited foresight.

Keywords Carbon price, Banking effect, Limited foresight, Markets with frictions, Optimization Model, Simulation model

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1 Introduction

Decarbonization requires massive, rapid investment in low carbon technologies, notably in the electricity sector. Yet related markets for risk sharing are notoriously acknowledged to be missing or at least incomplete, which impairs investment incentives and warrants the use of long-term contracts (Joskow, 2022; Keppler et al., 2022). This is notably recognized in the recent electricity market design reform in the EU (Fabra, 2023), which promotes contracts for difference and power purchase agreements. In turn, these contracts will increasingly interact with the carbon price in the EU carbon market, which is the main decarbonization policy in the EU.

These interactions and associated impacts on investment incentives, installed capacities, and contract properties deserve further analysis. In fact, impacts on installed capacities can be different in nature given that a carbon price is technology-neutral, whereas long-term contracts tend to be technology-specific. In principle, "perfect" carbon pricing could partly compensate for the absence of long-term risk markets for electricity (Dimanchev et al., 2024). However, in practice, the carbon market is also subject to various frictions, notably limited foresight (Quemin & Trotignon, 2021), while simulation models that formally represent the electricity and carbon markets with endogenous investment are few and typically assume "ideal" markets (Bruninx et al., 2020; Pahle et al., 2025). Moreover, the relatively higher capital intensity of low-emission technologies constrains their development, and existing modeling efforts indicate that investment becomes riskier - in turn, reduced and or delayed - as electricity price volatility tends to be higher with greater penetration of renewables (Jimenez Sanchez et al., 2024; Lebeau et al., 2024).

The objective of this paper is to examine these emerging trends with new contractual arrangements for electricity and "imperfect" carbon pricing. The main contribution is the development of a modeling framework with an endogeneous carbon price for studying investment dynamics and the role of carbon price in carbon and electricity markets subject to frictions.

The model used in this paper is composed of two main blocks that build on and extend Lebeau et al. (2024). The first block is a multi-year optimization problem that determines the optimal generating fleet, i.e., a Generation Expansion Planning (GEP) problem with a multi-year carbon constraint. The second block simulates investment and closure decisions of a representative market actor. This operates as a loop where, for each year, entry and exit decisions are made based on expected individual asset profitability using a gradient descent method. This

approach captures the iterative decision-making process of a typical investor and enables the incorporation of risk aversion (here, through certainty equivalents). The advantage of this two-block model is that the information transfer from the first block (representing a "perfect" market) and the second block (simulating an "imperfect" but more realistic market) can be varied. In particular, if-and only when-the simulation model is supplied with information in line with GEP outcomes, it can iteratively reconstruct the optimal fleet (Lebeau et al., 2024).

The modeling framework extends that of Lebeau et al. (2024) by endogenizing the carbon price (i.e., allowing it to respond to changes in installed capacity dynamics and associated emissions) with various degrees of responsiveness. In each part of the model different methods are used to ensure that the price of carbon and the price of electricity are co-determined. This modeling property is a key contribution of this paper.

Specifically in the first block, the carbon price is derived from the dual variables of the multi-year optimization problem, and two calculation methods are deployed depending on whether intertemporal flexibility in the carbon market (i.e., banking) is allowed or not. Anke et al. (2020); Van den Bergh et al. (2013) model an endogenization of the carbon price in an optimization model through a similar use of the dual variables with an annual constraint. The endogenization of the carbon price enables them to conclude on the destructive effects of the cohabitation of different climate policies with the carbon price. Our model extends this generic method to a less specific case by allowing the endogenization of the carbon price in an optimization model in the presence of a multi-year carbon constraint. The previous conclusions stays and our paper adds that the stability of the carbon price in the case of a more flexible system leads to lower system costs. Further our model also allows us to study the system's response to different distributions of emissions permits.

In the second block, the carbon price is determined by iterative search so that the emissions of the generating fleet meet the emission constraints. Richstein et al. (2015) use a similar method in an agent-based model. Their model uses load duration curve meanwhile our model includes an hourly dispatch. The use of an hourly dispatch allows to show the consequences of the degeneracy (Pierru, 2007) in the system's reactions to different regulations. In particular, it shows that in the presence of degeneracy, the equivalence between price and quantity policies is no longer valid. We consider that imperfections in the carbon market are reflected in the simulation model by a parameter capturing market actors' myopia (this parameter can be varied extending the method developped in Richstein et al. (2015)). Convergence of the

carbon price in the second block is helped by an heuristic based on the price behavior observed in the first block. This heuristic leverages differences in emission rates and generation costs between technologies and cumulative emissions as inputs. Inspiration of this method can be found in the impact curves of [Delarue & Van den Bergh \(2016\)](#).

The main results are threefold.

This paper studies how price and quantity-based regulations differ in a generation fleet of long-lived assets, that is, in the presence of adjustment costs and capital inertia ([Williams III, 2010](#)). The investment dynamics and overall emissions under an "equivalent" carbon market and tax are equivalent to the extent there is no degeneracy. We compare the nature and relevance of the information or signal conveyed by the dual variables (prices) compared to their primal constraints (quantities). The signals from the annual carbon budget constraint into prices imposed on each hourly variable cost. This transition from one time scale to another explains the sensitivity of equivalence to degeneracy. In the presence of several carbon assets involved in the fuel switch, then the emissions tend to be lower overall under a tax, even when there are no frictions in the carbon market.

Second, this paper shows that an intertemporal flexibility in emissions through banking in the carbon market leads to more stable carbon prices and sustained investment dynamics—with banking, short-run and long-run carbon prices coincide. In the absence of banking, we find that the price surge at the end is due to a lack of investments in the previous years, and it triggers needed investments at a "too" late stage. Banking allows firms to smooth compliance costs over time and dampen price volatility. Similar results can be found in ([Rubin, 1996](#); [Schennach, 2000](#)) (partial equilibrium) and ([Dubois et al., 2025](#)) (general equilibrium) in the form of reduced business cycle fluctuations and GDP volatility. [Vogt-Schilb et al. \(2018\)](#) and [Storrøsten \(2020\)](#) find that the levelized cost of conserved carbon is higher than the optimal carbon price. This paper shows that it depends on different banking strategies. Further, this paper extends results from [Anke et al. \(2020\)](#) by adding the ability to bank emission allowances and by reducing the one-off impact of an external climate policy on the price of carbon. Besides, we characterize how different paths for emission caps (enforcing the same overall emission target) affect carbon prices and induced investment dynamics with and without banking. For instance, we compare a "linear" cap as in the EU carbon market with a one-step staircase cap as in the sulfur trading system in the US, among other types of cap paths.

Third, this paper shows how frictions in the carbon market (i.e., limited foresight and price responsiveness) affect the rate of decarbonization and investment dynamics in the electricity market. Previous results of [Quemin & Trotignon \(2021\)](#) show that investment and banking decisions may also be affected by limited farsightedness. We find that a perfectly responsive carbon price is conducive to the optimal decarbonization path without the need for long-term contracts, even in the presence of limited foresight in the electricity and carbon markets (the cost of the delay induced by limited foresight is borne by the carbon price). A different approach conducting a similar result is given in [Dimanchev et al. \(2024\)](#) for risk-aversion. When actors exhibit myopia in the carbon market, their propensity to invest is diminished, leading to lower and delayed investment in low-carbon assets. This effect is increasing with the degrees of myopia and inertia in price responsiveness. Long-term contracts have the potential to compensate for this investment deficit and delay. The next step in our work is to study how contract properties (e.g., duration, shape) hinge on the degree and nature of imperfections in the electricity and carbon markets.

This paper is structured as follows: In section [2](#), an overview of the existing literature on energy modeling and on carbon pricing in energy models is presented. In Section [3](#) the hybrid model used in the paper is described and the different methods used to endogenize carbon price are detailed. Section [4](#) introduces the California study case used in this paper to find the results of Section [5](#). Section [6](#) concludes this paper with a summary of the core findings and policy recommendations.

2 Literature review

2.1 Modeling electricity markets

Various approaches have been utilized by energy economists and engineers to analyze and understand long-term power system challenges. These approaches can be broadly categorized into three types: optimization models, equilibrium models, and simulation models ([Ventosa et al., 2005](#)).

Optimization models Optimization models are the original and traditional approach to modeling the evolution of energy systems. The so-called generation expansion planning

(GEP) models typically take the perspective of a central planner that seeks to determine the socially optimal capacity development plan (i.e., that which minimizes system-wide investment and operating costs) given a variety of constraints (e.g., a cap on carbon emissions), see [Kagianas et al. \(2004\)](#) for a historical perspective. Over time, GEP models have notably been extended to stochastic frameworks and are still widely used today to analyze decarbonization pathways for energy systems, see [Weber et al. \(2021\)](#) for a recent review.

Equilibrium models Equilibrium models simultaneously solve individual profit maximization problems for different types of agents (e.g., producers with different technologies, intermediaries, consumers), finding equilibrium solutions where no agent is better off deviating unilaterally (e.g., [Fan et al. \(2012\)](#)) These models typically feature uncertainty and risk aversion (e.g., [Ehrenmann & Smeers \(2011\)](#); [Abada et al. \(2017\)](#); [Mays et al. \(2019\)](#); [Mays & Jenkins \(2022\)](#)) or imperfect competition (e.g., [Hobbs & Pang \(2007\)](#); [Acemoglu et al. \(2017\)](#)).

Simulation models Simulation models can represent different decision-making rules (i.e., beyond profit maximization) and degrees of agent’s sophistication and rationality. There are two broad types of simulation models: The first is agent-based modeling (ABM) that can feature heterogeneous agents. The second uses system dynamics (SD) and typically consider the whole system. [dos Santos & Saraiva \(2021\)](#) and [Tao et al. \(2021\)](#) (resp. [Teufel et al. \(2013\)](#) and [Ahmad et al. \(2016\)](#)) provide useful reviews of ABM (resp. SD models) applied to energy systems.

Optimization models do not take fully account of market realities, but they do provide a benchmark for changes in the electricity mix. They maximise social well-being and are therefore used to define public policy objectives. In this way, they can often be used to obtain forward-looking scenarios that are interesting for studying the future of investment in electricity mixes. However, they do not incorporate all the characteristics of market players, since they are based on neoclassical assumptions (rationality of agents, perfect information for instance). These shortcomings prevent them from accurately representing the decisions of market players in the face of market imperfections. They do not model the decision-making processes of market participants (possibly with bounded rationality, information or foresight) or the sequentiality of discrete investment/divestment decisions over time (since all time steps are solved simultaneously).

Equilibrium models allow modelers to represent and assess the impacts of the market structure and heterogeneous agents and behaviors. This notably endogenizes key decisions and model variables (e.g., risk trading and the associated cost of capital). Yet these models rely on solvers whose results do not lend themselves to a straightforward interpretation of the mechanisms leading to the equilibrium, and they are typically solved in steady state, which does not unveil the dynamic nature of investment/divestment decisions. Additionally, by design these models cannot account for out-of-equilibrium situations, which are acknowledged to be common and deserve more attention ([de Vries & Heijnen, 2008](#); [Léautier, 2019](#)).

Compared to equilibrium models, simulation models give more latitude in making explicit assumptions about agents' rationality, information and foresight levels, and in representing out-of-equilibrium situations. Arguably, this strength may also be a weakness, in that assumptions must be clearly spelled out and articulated with one another in order to arrive at sensible modeling results. Both ABM and SD models have been widely used, notably to study capacity markets and more recently other market design issues in the context of the energy transition.

The evolution of investment in the electricity mix is studied in relation to different public policies linked to decarbonization. The aim is to compare two tools used for decarbonization: a carbon tax (a price-based decarbonization tool) and quotas from which a price is derived (a quantity-based decarbonization tool). The added value of this publication is the study of these tools in the context of imperfect markets. The modeling used in this article makes it possible to study the behaviour of market participants in the face of these various public policies in a context of imperfect markets. The carbon price in the model is set endogenously as a function of changes in the underlying fleet.

The use of a MCP model to ensure endogenous carbon pricing makes it possible to show that, in the context of a perfect carbon market, decarbonization and security of supply are guaranteed [Dimanchev et al. \(2024\)](#). The equilibrium search inherent in this modelling does not provide access to the explicit functioning of a sequence of market decisions that is sought in the approach of this article. Additionally, the proper functioning of MCP resolutions is ensured for convex problem resolutions. The freedom of choice of assumptions in a simulation model makes it easier to integrate features without the risk of unfeasibility due to the non-convexity of the problem.

The chosen modelling framework is an hybrid structure developped in [Lebeau et al. \(2024\)](#). A first part is an optimization model and the second one uses the simulation framework. Details are given in [Section 3.1](#)

2.2 Carbon price in electricity markets

This paper connects and contributes to different strands of literature. Notably, it gives an informational and energy modeling optimization flavor to climate policies.

2.2.1 Climate policy and carbon pricing

Irreversible investment in decarbonized technologies by firms covered under a permit market is reduced and delayed in equilibrium as firm-specific or market-wide uncertainty increases, even for risk-neutral firms (Chao & Wilson, 1993; Zhao, 2003). Permit trading provides firms with more flexibility to adapt to shocks than investment does, which generates an incentive to wait before investing and entails that permit prices exceed (short-run) marginal abatement costs by some positive option value (Chao & Wilson, 1993; Taschini, 2021). This holds even when permits cannot be banked for future compliance. When permits can be banked, banking can act as a substitute for irreversible investment in permanent abatement and improved abatement technologies (Phaneuf & Requate, 2002; Slechten, 2013). This holds even in the absence of uncertainty, as firms can use both investment and banking to smooth their abatement costs over time. Bringing these two separate results together may suggest that in the presence of both uncertainty and banking, investment should be further reduced and delayed. But this need not be the case. In fact, Pommeret & Schubert (2018) show that the downside of irreversibility is in most cases mitigated by the additional flexibility provided by banking. Specifically, substitutability between banking and early investment in low-carbon capital breaks down when marginal abatement costs are convex. Net-zero target means substitution can only be temporary, also because banking will end (Pahle et al., 2025). Anticipating this, investment must occur early on. And in this case, because banking leads to higher price upfront through its phase-in property. Further, banking may also alleviate trading frictions (Baudry et al., 2021; Toyama, 2024).

The introduction of the EU ETS in 2005 and its subsequent reforms are the result of economic thinking on the role a carbon market can play in decarbonization. Given that the EU ETS system allocates permits in a decreasing manner, regulation provides an incentive to decarbonize means of production (Boehringer et al., 2016). Obligated players are thus faced with the choice of paying more and more for carbon quotas or decarbonizing their means of production (this applies to players in power generation, but also to heavy industry). Most permits are requested by power generation companies (EEA, 2024), which means that their banking strategy can have an impact on price formation. The price derived from the EU-

ETS market enables the merit-order to be modified in favor of less carbon-intensive means of production (Anke et al., 2020).

2.2.2 Bridge with energy market modeling

Studies of long-term electricity generation fleets often consider the price of carbon as an exogenous variable Weigt et al. (2013); Brink et al. (2016). However, incorporating an endogenous carbon price provides deeper insights into market dynamics and policy effectiveness. Below, we review some studies that have explored the implications of endogenous carbon pricing and other decarbonization policies using different modeling approaches.

The general equilibrium model developed by Brink et al. (2016) demonstrates the interest of a carbon price floor to ensure decarbonization using an endogenous carbon price with cross-border trading. Similarly, Chappin et al. (2017) and Richstein et al. (2015) use agent-based modeling to integrate an endogenous carbon price and a carbon price floor, highlighting its impact on investment decisions. These models show that setting a carbon price floor can stabilize market conditions and promote decarbonization efforts.

The simulation model used in Franco et al. (2015) reveals that a carbon market alone is insufficient to meet decarbonization and security of supply objectives. This emphasizes the need for additional policy measures to complement carbon pricing, such as long-term contracts and regulatory frameworks. Relatedly, Delarue & Van den Bergh (2016) and Delarue et al. (2007) study long-term investment with an endogenous carbon price and mandatory closures of carbon-intensive facilities and penetration of renewable assets through out-of-market policies, providing insights into efficient policy design. They show that if the emissions cap of a cap and trade is not adjusted to reflect the emission reductions induced by complementary policies, the effectiveness of the carbon price in promoting short-term decarbonization diminishes.

A limited banking mechanism for emissions permits is used by Richstein et al. (2015) with an agent-based model to study the investment dynamics. This approach underscores the importance of integrating banking into carbon pricing models to enhance market flexibility and therefore greater incentive to invest.

The methods developed in the present paper to endogenize the carbon price in a multi-year optimization model are a key contribution to the literature. This paper proposes a comprehensive approach to integrate both the banking component and the endogeneity of the carbon price, addressing gaps in existing literature. Specifically, our study focuses on

electricity and carbon markets characterized by frictions such as myopia and risk aversion, which, to the best of our knowledge, have not been fully covered in previous research.

3 Method and model

3.1 Model description

The proposed modeling framework extends that of [Lebeau et al. \(2024\)](#). It is a hybrid model that allows two different blocks to interact. The first block is a multi-year optimization problem that determines the optimal generating fleet, i.e., a Generation Expansion Planning (GEP) problem with a multi-year carbon constraint. The second block simulates the decisions of a representative market actor. It is based on the framework of simulation models. The objective of this bloc is to determine the behaviour of a representative agent in the system. To do so, it operates as a loop where, for each year, investment decisions are made based on expected individual asset profitability using a gradient descent method. This approach captures the iterative decision-making process of a typical investor and enables the incorporation of risk aversion (here, through certainty equivalents). The advantage of this two-block model is that the information transfer from the first block (representing a 'perfect' market) to the second block (simulating an 'imperfect' but more realistic market) can be varied.

This modeling makes it possible to compare the investment trajectories from the two blocks to show the differences between perfect markets and markets with frictions. The use of a simulation model as the second block of this framework also provides greater control over the assumptions made by agents when making decisions that are relevant to investment choices (risk aversion and agent myopia). Further, it is possible to vary the friction assumptions in the simulation module so as to observe the consequences of these variations on the rate of investment and the cost of the system. The use of a representative player makes it possible to obtain results at an aggregated level without having to model each player in the electricity mix separately. As mentioned previously, this modeling allows greater freedom in the assumptions made by the agents, but in return, it is necessary to pay close attention to them. Simulation model results are sensitive to (long-term) price projection methods as noted by ([Tao et al., 2021](#)) and ([Fraunholz et al., 2021](#)), and in particular to the way future capacity developments are anticipated and impact future price formation - and in turn govern investment decisions.

The interactions between the two simulation blocks are sequential. The first block runs first with the input data. The GEP problem is solved according through the formulation given in Appendix B. The GEP then provides: the investment trajectory and CO₂ prices in a given environment (e.g. absence of risk-averse actors, perfect foresight). Obtaining CO₂ prices is detailed in Section 3.2 and depends on the choice of decarbonization policy studied. The use of 'perfect' investment trajectories is twofold: they are both the benchmark against which it will be possible to compare trajectories resulting from a market with friction and the basis for certain hypotheses of the simulation model in the second block. The details of these dependencies are presented later in the article.

3.1.1 Optimization - GEP

The GEP problem is written as a constrained investment cost minimization model. The minimisation problem is given in Appendix B and details are given in Lebeau et al. (2024). In the GEP, the regulator has several tools at its disposal to ensure the decarbonization of electricity generation. The first is to introduce a carbon tax, which in the model takes the form of an initial carbon price (within the variable costs of carbon assets) and without additional constraints. The second tool is to introduce an emission volume constraint, which takes the form of an initial carbon price of zero and a carbon budget constraint. This constraint can be of two kinds: an annual constraint and a multi-year constraint. Either the regulator sets the player a target per year that must be met, or it sets a multi-year volume target that may enable the player to establish its own decarbonization strategy. The banking strategy for emission permits that may then result is the idea of decarbonizing more at the beginning of the period in order to have more emission permits available when they are scarcer at the end of the period.

Carbon Tax If we add a carbon tax, in the model we end up with a CO₂ price $\pi_y^{CO_2}$ like a commodity price which is added to the price of fuel $Fuel_{t,y}$ consumed by the technology.

$$VC_{y,t} = \frac{Fuel_{t,y}}{\eta_t} + \zeta_t \cdot \pi_y^{CO_2} \quad (1)$$

Carbon budget If the regulator decides to apply a policy on the volume of emissions issued by players, the added constraint in the GEP problem is as follows without intertemporal

flexibility:

$$\forall y \in \mathcal{Y}, \quad \sum_{w \in \mathcal{W}} \Pi_w \sum_{t \in \mathcal{T}} \sum_{h \in \mathcal{H}} \zeta_t \cdot q_{t,y,w,h} \leq Q_y \quad (\gamma_y) \quad (2)$$

When players have intertemporal flexibility and can store emission permits for later use, the constraint then becomes :

$$\forall y \in \mathcal{Y}, \quad \sum_{k=0}^y \sum_{w \in \mathcal{W}} \Pi_w \sum_{t \in \mathcal{T}} \sum_{h \in \mathcal{H}} \zeta_t \cdot q_{t,k,w,h} \leq \sum_{k=0}^y Q_k \quad (\bar{\gamma}_y) \quad (3)$$

Each constraint is associated with a dual variable γ in the case of an annual constraint and $\bar{\gamma}$ in the case of a multi-annual constraint. These two constraints produce indeed different dual variables. From these dual variables it is possible to derive a price that corresponds to the underlying carbon market. Details of the method used to obtain the price from the dual variables are given in Section 3.2. These prices reconstructed by the dual variables are then considered as the 'perfect' carbon prices introduced as output of the optimization module and input of the simulation module.

3.1.2 Simulation - Merchant Loop

The simulation model behaves like a loop over the years. At each year, the system imports the GEP's investment choices for the next years (up to the end of the simulation horizon) via an anticipation module, and builds a long-term scenario (with other elements such as commodity prices and future demand). For each year, the system then tests each asset whose addition is endogenous. To determine the profitability of the investment, the dispatch then uses the previously created long-term scenario. When all assets have been tested (including divestment choices), the system selects the one with the highest NPV. Once this first asset has been invested in the current year, the system retests all assets using the gradient descent method. When no additions or deletions are profitable (the gradient descent has converged), the system checks whether the CO2 emissions respect the distributed quotas. If the system respects the quotas, then the simulation loop continues with next year. Otherwise, the loop repeats all asset tests for year y , with a different CO2 price.

The simulation model is built upon the framework introduced in (Lebeau et al., 2024). Enhancing this foundation, a price-feedback loop for carbon pricing is integrated into the simulation module, and a multi-year carbon constraint is calibrated within the optimization module.

3.2 Carbon price definition methods

The endogenous nature of the carbon price is at the heart of the added value of this paper. The co-determination of the price of carbon and the price of electricity in the case of a study of long-term investments is the objective of this section.

First, the determination of the carbon price in an optimization model is studied. In the optimization model, our method uses dual variables. In particular, [Poncelet et al. \(2019\)](#) has demonstrated that, under specific conditions, the dual variable associated with an annual linking constraint can be interpreted as a shadow price. Furthermore, [Aucamp & Steinberg \(1982\)](#) elucidates the fundamental distinctions between shadow prices and dual variables, and explores the implications of interpretations that circumvent the mathematical foundations inherent in optimization problems. Our approach does not aim to diverge from this framework but rather presents an interpretation of a particular transformation of the dual variables of the carbon constraint as a CO₂ price which can be used as in (1). The objective here is not to mathematically establish the equivalence of this interpretation with the traditional definition of a shadow price, which corresponds to the right-hand derivative of the optimal cost function under the carbon constraint. Instead, we focus solely on a demonstration of the properties of this transformation of dual variables.

Second, the determination of carbon price in a simulation model is studied. The development of an endogenized carbon price allows a simulation model to apply a restoring force in the event of deviation from the optimal trajectory to ensure decarbonization. It is an extension of [Lebeau et al. \(2024\)](#). Indeed, if the system makes decisions that do not allow emissions quotas to be respected, then the carbon price will be higher to encourage more decarbonized investments. Iteratively evolving the carbon price in a simulation model is achieved by [Richstein et al. \(2015\)](#) and allows us to observe the evolution of low-carbon investment dynamics. Hybridization between the two modules could make it possible to create a GEP for each year and re-implement carbon price trajectories that would perfectly make up for a shortfall or excess of investment made in the gradient descent of the investment module. However, beyond the rather high computational cost, the properties of a GEP would not match the need. A GEP catches up or corrects its lead from the very first year (this is true for the production fleet and therefore for the corresponding carbon price). Using the GEP as a simple update would prevent a true propagation of information between years - which is allowed by the method developed by this article and which corresponds more to a market reality (than that of a one-off catch-up on the next year of the horizon).

3.2.1 Optimization module

In the case of the optimization problem, as presented in Section 3.1.1, either the regulation imposes a CO₂ price level, or a volume through a carbon constraint. To be able to find the prices resulting from the constraints, the model method uses the dual variables associated with these constraints.

Annual constraint A price can be derived from the dual variable of an annual and linking constraint. As (Poncelet et al., 2019) explains, *"each linking constraint integrated in a surplus maximization problem will thus indirectly describe a market, i.e., both the price (dual variable of the linking constraint) and the variables receiving/having to pay this price are indirectly specified via the linking constraints."* Like the demand-satisfaction constraint, the carbon constraint operates as a linking constraint in this optimization model, with the difference that the carbon constraint incorporates a multi-step-time dimension. In the annual case, the carbon constraint imposes a quota on all time steps of the same year.

$$\forall y \in \mathcal{Y}, \quad \pi_y^{CO_2} = -\gamma_y \cdot \beta^{-y} \quad (4)$$

The dual variable associated to the carbon constraint of the year y is thus interpreted as a cost avoided by decarbonization. It can be interpreted as a carbon price. In fact Appendix C.1 we demonstrate that the equivalent carbon tax of a system where a volume emission constraint is imposed is indeed the opposite of the dual variable associated with this carbon constraint. Mathematical operations provided in Appendix C.2 enable us to express the content of this dual variable. Indeed, it appears to be a ratio between a fixed-variable cost trade-off and a difference in emission rates.

For instance, if we consider a system with two technologies, it is possible to find the equilibrium carbon price. It is the price that allows the fuel switch in a generating fleet. In the case studied here, this means that we are trying to express π^{CO_2} such that :

$$VC_{y,1} + \zeta_1 \cdot \pi_y^{CO_2} = VC_{y,2} + \zeta_2 \cdot \pi_y^{CO_2} \quad (5)$$

It comes :

$$\pi_y^{CO_2} = \frac{VC_{y,2} - VC_{y,1}}{\zeta_1 - \zeta_2} \quad (6)$$

In the case studied, it is also possible to write the dual variable of the carbon constraint. It

gives :

$$\gamma_y = \beta^y \frac{VC_{y,1} - VC_{y,2}}{\xi_1 - \xi_2} \quad (7)$$

Then one can write :

$$\pi_y^{CO_2} = -\beta^{-y} \cdot \gamma_y = \frac{VC_{y,1} - VC_{y,2}}{\xi_1 - \xi_2} \quad (8)$$

In such case, finding the equilibrium carbon price enables to find the exact composition of the dual variable. Economic reasoning is thus found when we interpret this transformation of the dual variable as a price. For instance; with fixed emission rates, if the two technologies have similar costs, then it is easy to switch from one technology to the other: the system can move more easily towards the low carbon technology. With fixed costs, if the two technologies have close emission rates, the gains from switching from one technology to the other to decarbonise will be low: it will be more difficult for the system to move towards the decarbonised technology.

Pluriannual constraint In the case where emissions banking is authorised by the regulator - i.e. when the system can store permits from one year for use in a later year - the dual variables associated with the carbon constraints are no longer identical to those used in (4). Their interpretation as a carbon price must also be modified so that it remains consistent. Indeed, if the system decides not to bank its emissions permits, then the carbon price interpreted from the dual variables must converge with that interpreted in the case of an annual constraint. The formulation proposed in this article (and corresponding to our optimization problem) is as follows (see Appendix for the details):

$$\forall y \in \mathcal{Y}, \quad \pi_y^{CO_2} = -\beta^{-y} \cdot \sum_{k=y}^{\#\mathcal{Y}} \bar{\gamma}_k \quad (9)$$

Two different elements need to be verified to confirm the interpretation of this transformation of dual variables into prices. The first is to check that this formulation coincides with the one in (4) with a system whose constraints allow it to bank its emission permits and whose parameters do not allow it to bank (i.e. discount rate too low, carbon target too restrictive). This first element is well verified (see Appendix for details) and we can write: $\forall y \in \mathcal{Y}, \gamma_y = \sum_{k=y}^{\#\mathcal{Y}} \bar{\gamma}_k$. The second element to check is that if the system decides not to issue all its quotas during a period of time, then the price must evolve according to Hotelling's rule ([Quemin & Trotignon, 2021](#); [Anke et al., 2020](#)). This is verified by the second-order conditions (see Appendix C.2 for details).

The interpretation of the CO₂ price presented at the beginning of Section 3.2.1 is verified in the generic case (see Appendix C.3) to the extent it is possible to write without a too high loss of generality:

$$\forall y \in \mathcal{Y}, \forall t \in \mathcal{T}, \quad k_t \cdot IC_{y,t} \cdot \sum_{i=y}^{\#\mathcal{Y}} \beta^i = \sum_{i=y}^{\#\mathcal{Y}} \sum_{h \in \mathcal{H}} q_{i,h,t} \cdot (\lambda_{i,h} - VC_{i,t} + \zeta_t \cdot \gamma_i) \quad (10)$$

$$\forall y \in \mathcal{Y}, \forall t \in \mathcal{T}, \quad k_t \cdot IC_{y,t} \cdot \sum_{i=y}^{\#\mathcal{Y}} \beta^i = \sum_{i=y}^{\#\mathcal{Y}} \sum_{h \in \mathcal{H}} q_{i,h,t} \cdot (\lambda_{i,h} - VC_{i,t} + \zeta_t \cdot \sum_{j=y}^{\#\mathcal{Y}} \gamma_j) \quad (11)$$

The two equations (10) and (11) show the recovery of investment costs in the system for a technology, allowing us to write that the inframarginal income of each hour over all the years finances the investment costs. This inframarginal income is subtracted from the technology's CO₂ cost.

3.2.2 Simulation module

In the simulation module in our modelling framework, carbon price is determined iteratively :a loop on a carbon budget enables an intergration of a quantity-based regulation.

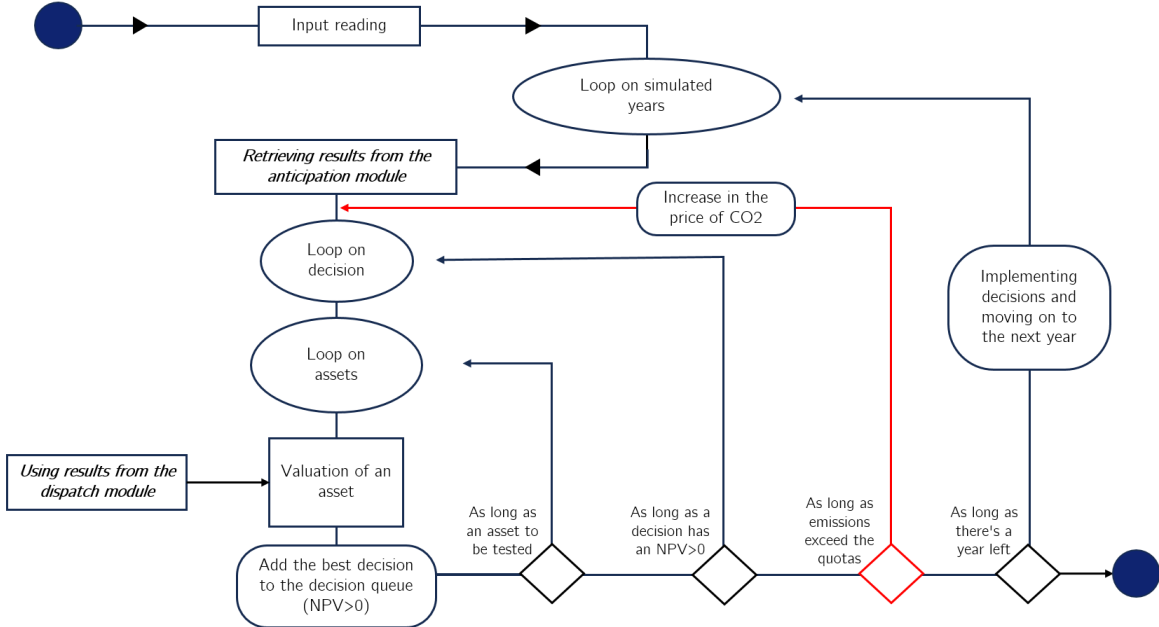


Figure 1: Detail of the loops in the decision module

Carbon price loop The decision module in the simulation module proceeds through several nested loops with a gradient descent (Lebeau et al., 2024) to obtain the best decisions for the system. Before carrying out the successive loops, it uses the results of the anticipation module, which provides it with long-term investment trajectories, commodity prices and the price of CO₂. The carbon price is then either taken from the results of the optimization model in order to maintain price regulation, or it is supplemented by a loop on CO₂ emissions, which ensures that the model continues to pursue the decarbonization policy. When a loop over a year is about to end - i.e. when all the profitable assets have been simulated and selected for construction thanks to the gradient descent algorithm - then the system carries out a virtual dispatch of the future new generating fleet and measures the expected carbon emissions. If the carbon emissions exceed the quotas imposed by the regulator, then the carbon price is increased and the various asset profitability test loops continue again. The criterion for stopping the loop is to respect an interval around the distributed quota or a number of iterations.

In the case of an annual constraint, when the system simulates a year then it measures the carbon emissions of the years after the current one up to the end of its visibility horizon (which is either the end of the simulation horizon, or the one depending on its myopia if we decide to test this parameter - see Section 5 for the different study cases).

Depending on the choice of regulation concerning the use of allowances: annual or multiannual, the shutdown criteria and the CO₂ increase phases are different.

In both case, decision criteria for the carbon price loop are the GEP's annual constraints (2) (or (3)). With an annual carbon budget constraints, if one of the constraints is not respected, the carbon price of the relevant year is increased to need (see the charts drawn from the price elasticities). With a pluriannual constraint : either there is an unrespected constraint then the carbon price of the current year is increase to need and the other years' prices are increased by the interest rate (according Hotelling's rule). Either all the years are under the emission target, then the carbon price is decreased.

Diagram of elasticity To ensure faster convergence, the increase or decrease in carbon prices during the loop described above follows graphs of carbon price elasticity - CO₂ emissions. Two definitions of carbon price elasticity with respect to CO₂ emissions are studied.

- **Short-term elasticity:** when production assets are fixed, it defines the sensitivity of CO₂ emissions to carbon prices in the optimization model. An increase (resp. decrease) in the carbon price causes the dispatch to move towards more (resp. less) carbon-free

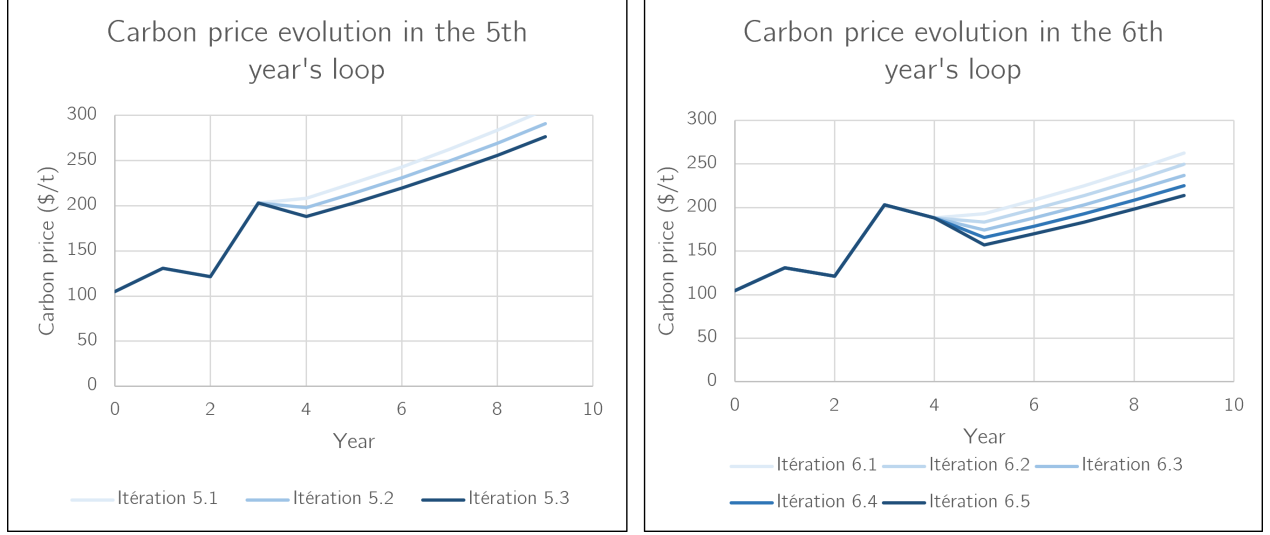


Figure 2: Evolution of carbon price between two years's loop

technologies.

- **Long-term elasticity:** measures the sensitivity of CO₂ emissions to the price of CO₂ without fixing the stock of power generation assets. In year y , if the carbon price encourages investment in a less carbon-intensive technology, this increases the long-term elasticity.

More formal definitions are given below. The variables involved are ϵ_{st} representing the short-term elasticity, ϵ_{lt} for the long-term elasticity, $Emissions_y$ are CO₂ emissions induced by the dispatch of the year y , $Emissions_{y+1}$ are the CO₂ emissions induced by the dispatch of year $y+1$ once investments decided in year y built and δ is the operator for a small local variation.

$$\epsilon_{st} = \frac{\delta Emissions_y}{\delta \pi_y^{CO_2}} \quad (12)$$

$$\epsilon_{lt} = \frac{\delta Emissions_{y+1}}{\delta \pi_y^{CO_2}} \quad (13)$$

The long-term and short-term elasticity diagrams are used as abacuses to speed up the convergence of the simulation module into the model presented in Section 3.1. If emissions in the current year of simulation (resp. future years) do not meet quotas, then the short-term (resp. long-term) elasticity chart is used. If elasticity diagrams are not usable, depending on simulation parameters, CO₂ price increments can be simply proportional to the differentials between actual emissions and emission allowance targets or can be a constant increasing or decreasing (+/- 10%).

4 Case study

The case study is based on a Californian dataset. The aim of the study is to observe investment and decommissioning decisions and the associated carbon price trajectories. To achieve this, simplifying assumptions are made. Firstly, although nodal pricing currently running in California substantially modifies price formation on wholesale markets, it is simplified here by considering zonal pricing with a single price zone. In addition, the system is assumed to be a single node (copper plate assumption), so network effects are outside the scope of the study. The data used are detailed in [Lebeau \(2023\)](#). The model thus seeks to develop asset inflows and outflows over the time period studied: here between 2025 and 2045. For reasons of model computation time, all the assets present in the generating fleet are not modeled individually, but only on an aggregated scale by technology. In addition, only certain technologies are endogenous: solar panel generation, battery storage, combined-cycle gas power plants and gas peakers. The other technologies are exogenous, and their trajectories are assumed to be either constant or based on CAISO projections (depending on the technology). Demand parameters are also exogenous and fixed in the model. Two demand scenarios are included for each year (as are 2 scenarios for the availability of renewable energies). One scenario is representative of average conditions with a 90% probability, and the other is representative of the one-in-ten peak with a 10% probability.

Low-carbon policy Given that our study focuses on the reaction of an electrical system to different public policies, we have to deal with different mechanisms. The first mechanism is the distribution of annual emission quotas. These quotas then form the parameters of the constraints introduced in [3.1.1](#). This mechanism models the operation of an ETS. Since the trajectory imposed by the regulator can have an impact on optimal investment trajectories, three major trajectories are studied. The first consists of a linear decrease in the distribution of emission allowances. This trajectory corresponds to an identification with what can be done in Europe (or California) for the CO₂ allowance distribution policy. A second trajectory consists of a staircase between two levels of allowance distribution. This method has been implemented in the USA in the SO₂ market. Finally, a third method is also used in this article: it lies between the two previous definitions. It proposes an annual reduction in distributed quotas, but this reduction is not constant: the reduction between two years at the beginning

of the horizon is smaller than that requested at the end of the horizon. This third trajectory is later called the linear-accelerated carbon budget. All these three carbon budget trajectory represent the same amount of carbon allowances : only the time distribution differs. Figure 3 show the different distributions.

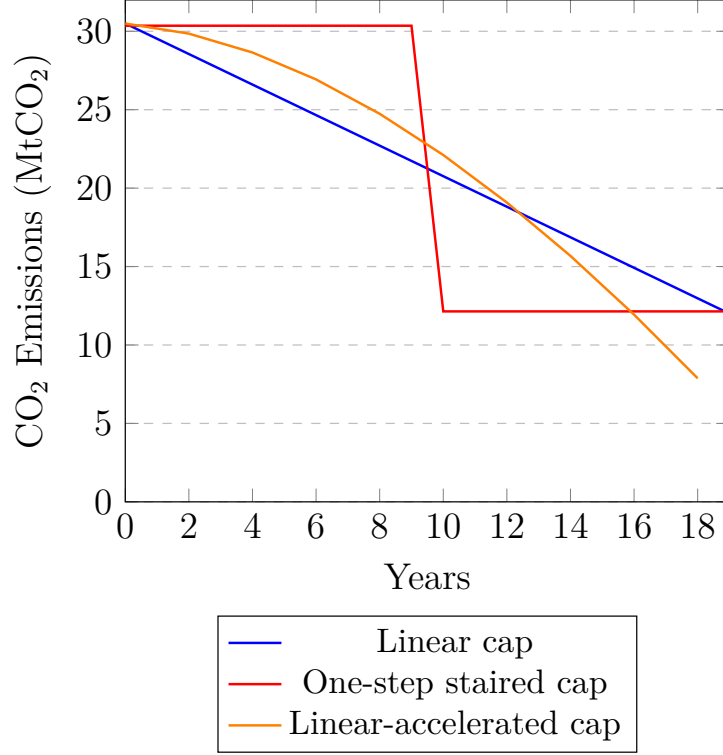


Figure 3: Studied carbon budget distributions

About time scope and solver The study focuses on a twenty-year period between 2025 and 2045. For some simulations, calculations are extended to 2050. As the highly computational difficulty of a simulation module, and in order to maintain an economic analysis based on a dispatch with hourly resolution, in some cases the choice is made to simulate one year out of two and to settle for a ten-year simulation. The simulation module converges for these ten years at around 45 hours of computing time. The solver used to solve the dispatches is CPLEX on an Intel®Xeon®Gold 6140 Processor (36 cores).

Initial generating fleet The modelling used in this article is based on an existing generating fleet (with its constraints and parameters), and seeks to adapt it as the simulation progresses. The generating fleet studied is composed of: 11.4 GW CCGT, 19 GW gas peakers, 10 GW battery, 2 GW Combined Heat and Power, 0.5 GW Nuclear Power Plants, 2.5

GW hydroly, 27 GW of solar pannels, 13.5 of wind turbines, 4 GW of other renewables energy sources (biomass, small hydro and geothermal). See [Lebeau \(2023\)](#) for details.

5 Results

This section compares the economic performance of the different tested situations. To do so, we compare the trajectories of endogenous investments (solar, storage and gas peakers) as well as CO₂ emissions and associated carbon prices. The minimum reference case tested is the one related with a regulation allowing full flexibility in the use of emission permits supplied to the system (e.g. a system allowing banking and borrowing of emission permits). A global carbon budget constraint leads to a perfect Hoteling’s rule for carbon price and it leads to the lowest system cost.

5.1 The effects of price- vs. quantity-based policy

First, we study the equivalence of price-based and quantity-based carbon policies facing different carbon budget flexibilities, different carbon budget paths and different generating fleets.

Banking effect The case of the generating fleet presented in Section 4 is used with a linear carbon budget cap.

- If the system is subject to a carbon constraint with no intertemporal flexibility (or so-called annual carbon constraint), then the difference between a price-based and a quantity-based carbon policy is less than $2.195e^{-4}\%$.
- If the system is subject to a carbon constraint with a perfect intertemporal flexibility (allowing borrowing and banking of the emission permits), then the difference between a price-based and a quantity-based carbon policy is less than $1.9e^{-11}\%$.
- If the system is subject to a carbon constraint with a partial intertemporal flexibility (only allowing banking of the emission permits), then the difference between a price-based and a quantity-based carbon policy is less than $2.187e^{-4}\%$.

For a given generating fleet and a given emissions permit trajectory, the equivalence between a price-based and a quantity-based carbon policy is ensured even facing different intertemporal

flexibility policies.

Carbon budget path effect The case of the generating fleet presented in Section 4 is used with a cross comparison between different carbon budget paths and different banking policies.

- When no intertemporal carbon flexibility is allowed, the difference between a price-based and a quantity-based carbon policy is less than $2.195e^{-4}\%$ (with a linear carbon budget cap), $8.111e^{-8}\%$ (with a one-step staired carbon budget cap) and $1e^{-10}\%$ (with a linear-accelerated carbon budget cap).
- When banking emission permits is allowed, the difference between a price-based and a quantity-based carbon policy is less than $2.187e^{-4}\%$ (with a linear carbon budget cap), $5.24e^{-9}\%$ (with a one-step staired carbon budget cap) and $1e^{-12}\%$ (with a linear-accelerated carbon budget cap).

For a given generating fleet, the equivalence between a price-based and a quantity-based carbon policy is ensured even facing different carbon budget paths.

Generating fleet effect If the generating fleet presented in Section 4 is modified to include more carbon-based means of production, the equivalence between a price-based and a quantity-based carbon policy is lost. Our preliminary analysis is that the addition of carbon-based electricity generation technologies multiplies dispatch optimum leading to degeneracy. In the case of quantity-based regulation, the system focuses on the physical distribution of CO₂ emissions, whereas in the case of price-based regulation, price arbitrage is independent of actual physical emissions. The consequence of degeneracy is the non-uniqueness of the cost minimum obtained by the GEP. In this case, the emissions associated with each of the minimums have no reason to be identical. Future work will focus on further characterizing generating fleets where the equivalence is lost.

The case of the generating fleet presented in Section 4 is used in the following subsections.

5.2 Effects of carbon budget paths

Using different carbon budget paths leads to different system costs. Here we study the system when no intertemporal flexibility on carbon emission permits is allowed.

- Facing a linear carbon budget path, the system cost is increased by 0.38% compared with the total system cost facing full intertemporal carbon flexibility.
- Facing a one-step staired carbon budget path, the system cost is increased by 2.15% compared with the total system cost facing full intertemporal carbon flexibility.

The greater the distance between the emissions imposed on a studied system and the use of emission carbon permits in a system with a carbon policy allowing a global carbon budget allocation, the greater the cost of the system.

5.3 Effects of banking

Adding temporal flexibility to CO₂ emissions in an annually constrained optimization problem leads to a more stable carbon price and lower system costs. The volatility of the carbon price is defined as the mean of the absolute error with the carbon price hired from the case of a global budget constraint. A high stability lead to a low mean absolute error.

Low volatility of carbon price

- In a system facing a linear carbon budget cap, if no carbon emission permits banking is allowed, then the carbon price volatility is about 6.22 %. When banking only is allowed, then the carbon price volatility is decreased to 5.17 %. Allowing carbon emission permits banking enable to reach 16.87 % of the maximum stability of the carbon price.
- In a system facing a one-step staired carbon budget cap, if no carbon emission permits banking is allowed, then the carbon price volatility is about 29.69 %. When banking only is allowed, then the carbon price volatility is decreased to 0.54 %. Allowing carbon emission permits banking enable to reach 98.17 % of the maximum stability of the carbon price.

The greater the distance between the carbon budget path imposed on a studied system and the use of emission carbon permits in a system with a carbon policy allowing a global carbon budget allocation, the greater the volatility of carbon price. Using a banking only policy enables to get significantly closer to the optimal case.

Interpretation of banking period We propose now to study the system when banking is allowed and how an interpretation of the banking period can be drawn from our results. First, three situations are possible concerning banking periods.

- The system increases the number of banked carbon emission permits. In such period, the system withdraws emission permits from the carbon market, driving up the price of carbon. This phase is notably present between years 8 and 14 when the system is faced with a linear distribution of allowances, and between years 0 and 10 when the system is faced with a one-step staired cap (see 4).
- The system decreases the number of banked carbon emission permits. In such period, the carbon price is lower than in a case where the system is not given any flexibility on intertemporal carbon permit allocation. This behavior can be seen between years 14 and 20 when the system is faced with a linear carbon budget. It is also found between years 10 and 20 for a system facing a one-step staired cap.
- The system neither increases or decreases the number of banked carbon emission permits. Then it converges well with the case where the system faces an annual carbon budget constraint with no possibility of banking. This property - demonstrated theoretically in the Appendix B - is illustrated with the case where the system faces a linear quota trajectory. The system's parameters make it adhere to the same emissions trajectory between years 4 and 8.

System costs

- In a system facing a linear carbon budget cap, if no carbon emission permits banking is allowed, then the total system cost is about 0.38% higher than the minimum with a full flexibility on carbon emission permits allocation (banking and borrowing).
- In a system facing a linear carbon budget cap, if only carbon emission permits banking is allowed, then the total system cost is about 0.35% higher than the minimum with a full flexibility on carbon emission permits allocation (banking and borrowing).
- In a system facing a one-step staired carbon budget cap, if no carbon emission permits banking is allowed, then the total system cost is about 2.15% higher than the minimum with a full flexibility on carbon emission permits allocation (banking and borrowing).

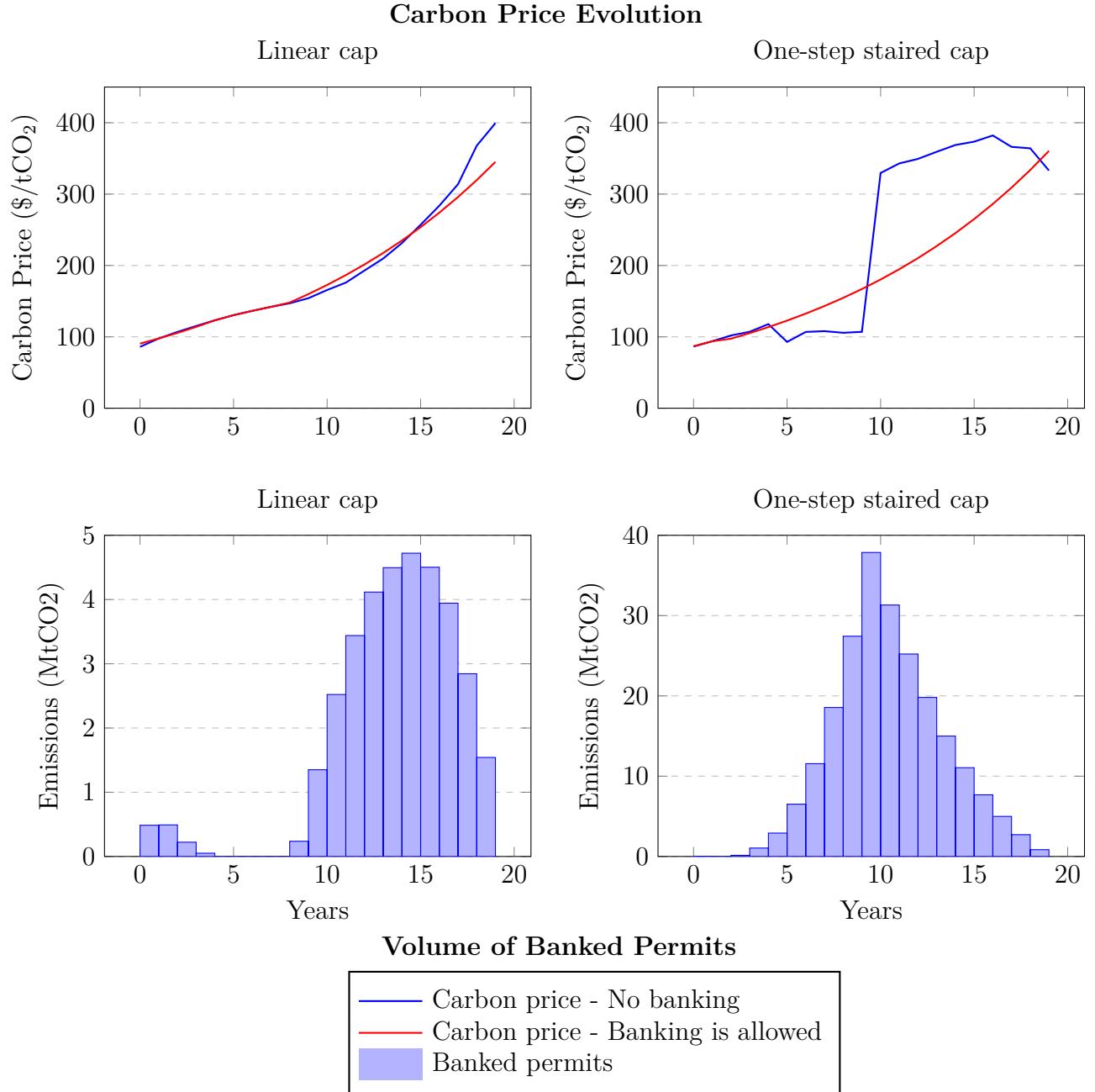


Figure 4: Carbon price evolution and volume of banked permits under different scenarios.

- In a system facing a linear carbon budget cap, if only carbon emission permits banking is allowed, then the total system cost is about 0.12% higher than the minimum with a full flexibility on carbon emission permits allocation (banking and borrowing).

When banking only is allowed, then the carbon price volatility is decreased to 5.17 %. Allowing carbon emission permits banking enable to reach 16.87 % of the maximum stability of the carbon price.

- Facing linear distribution of quotas : with banking : 5.61 % of the gain permis par la flexibility d'un full flexible par rapport \tilde{A} un annuel. A banking system is 0.35 % higher cost than a full-flexible. When a annual system is 0.38 % higher.
- Facing 1-step staired cap with banking : 94.41 % of the gain permis par la flexibility d'un full flexible par rapport \tilde{A} un annuel. A banking system is 0.12 % higher cost than a full-flexible. When a annual system is 2.15 % higher.
- Use of a banking policy enable the system to approach the solution given by the global budget constraint.

The use of banking enable the system to approach the optimal solution given with a global carbon budget constraint with a gain of 5.61% (resp. 94.41%) of the maximum cost gain in a system facing a linear cap (resp. a one-step staired cap).

Sensitivity analysis

- Impact of discount rate : banking is decreased if the interest rate is increased. Figure 5 shows the order of magnitude of the influence of the discount rate.
- Impact of the slope of a linear distribution : banking is increased if the slope is increased. Figure 5 shows the order of magnitude of the influence of the slope.

5.4 Effects of limited foresight

Our modeling allows specific assumptions to be isolated in the simulation module to determine their impact on decarbonization trajectories. The work of [Lebeau et al. \(2024\)](#) has already highlighted the influence of some frictions in an energy only market. We complement this approach here with special consideration for limited foresight in the carbon market. Other frictions, such as risk aversion, will be included and addressed in a later version of this work.

Dealing with different limited foresight First, the hybrid model in our study allows us to observe the consequences of limited foresight in different ways. Two different limited foresights can be observed in our simulation module.

- Limited foresight in electric market : it limits the system's rational anticipations in the computing of the NPV in the investment loop. With a time horizon limited to 5 years,

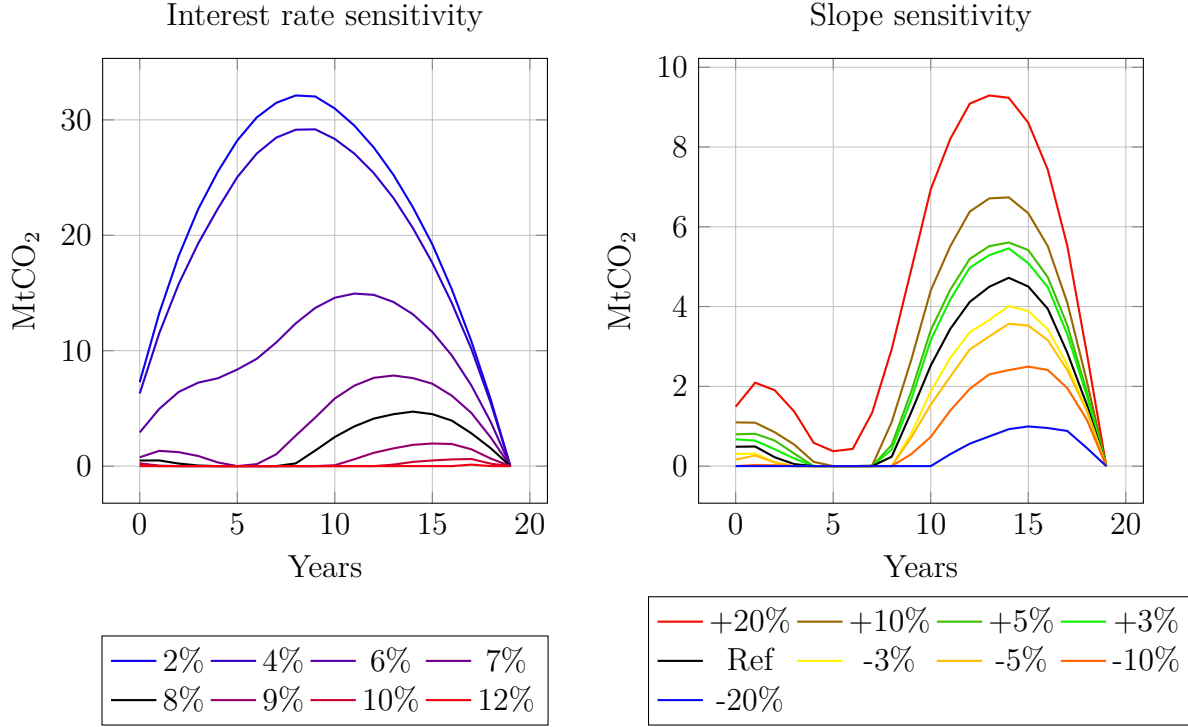


Figure 5: Banking trajectories facing parameters variations

the system makes investment decisions by considering the possible futures of the next 5 years only. It cannot take into account commodity price variations or availability beyond 5 years. The system does, however, allow him to invest, as the decision is made by considering the annuities corresponding to the investment (e.g. if the asset to be invested has a lifespan longer than the foresight, its profitability is in all cases calculated pro-rata to the lifespan via a transformation into annuities (Lebeau et al., 2024)).

- Limited foresight in carbon market : it limits the system's foresight on the carbon constraint in the carbon price loop. With a time horizon limited to 5 years, the system makes investment decision thanks to the gradient descent and check if the emissions of the next five years respect the carbon constraint. If the carbon constraint is not respected the 6th year for instance, the system does not take it into account when it computes the evolution of the carbon price.

In our optimization module, the time horizon of the GEP can also be varied. It enables us to observed consequences of limited foresight. By varying the GEP's time horizon, a rolling-horizon study can be done. (Quemin & Trotignon, 2021). Figure 6 depicts the method to iteratively build an investment trajectory with a GEP with a rolling horizon (3 years in the

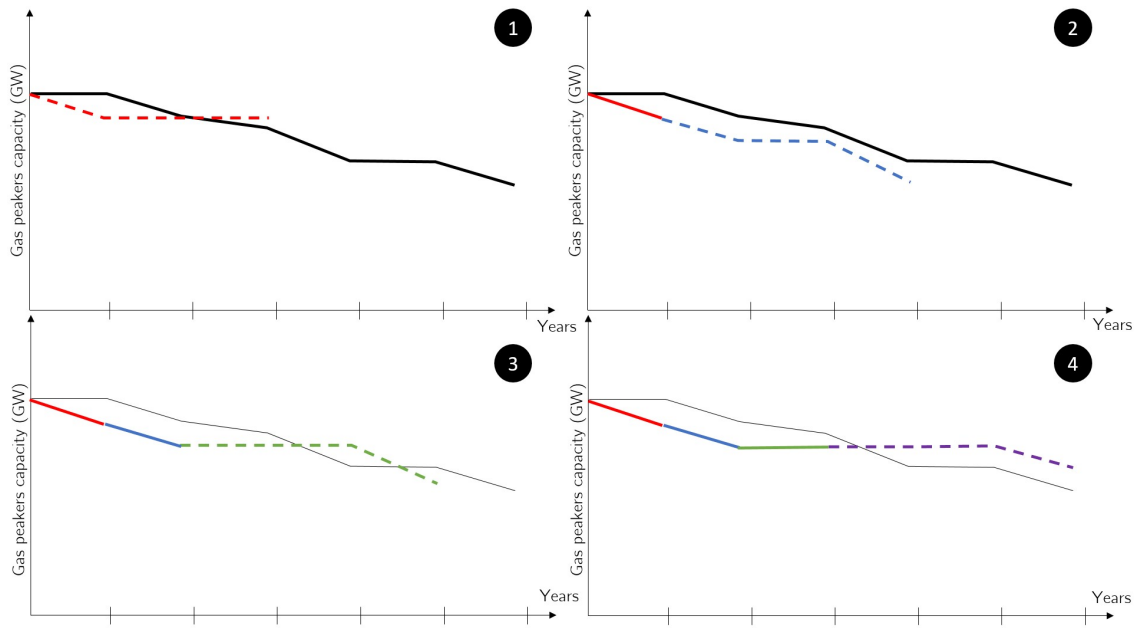


Figure 6: Evolution of gas peakers in a rolling horizon study (by the GEP)

figure).

But the GEP has only one time horizon (for electrical anticipation and for carbon constraint) so it is not possible to separate both limited foresight. A carbon market design effect is the only information which can be isolated thanks to this method. It consists in a comparison of different banking policy facing limited foresight of the same carbon budget path. The difference between the results can be used as a study of the carbon market design effect.

Carbon market design effect

- Facing a linear-accelerated carbon budget path and an annual carbon constraint (no intertemporal carbon emission permits flexibility), our system presents no variation between a GEP with a perfect foresight and a GEP with rolling horizon (neither in total cost or in investment trajectories).
- This previous result is lost when higher variations on demand, variable costs or investment costs are imposed in the system.
- Facing a linear-accelerated carbon budget path and an pluriannual carbon constraint (banking is allowed), our system presents some variations in total costs and investment trajectories between a GEP with a perfect foresight and a GEP with rolling horizon.

- These difference are only due to the carbon market design effect (because such differences do not exist in a simulation where everything is equal but the carbon market design).

This study enables us to observe the costs of the system.

- Our reference costs is the one reached by the system with banking and borrowing with perfect foresight. We study a system facing a linear-accelerated carbon budget path in perfect foresight. If banking is allowed then, it enables to reach 82.2% of the difference between the least favorable cost (with no intertemporal flexibility) and the optimum cost (banking and borrowing is allowed). Indeed, no bankin
- If a 3-years limited foresight is applied in the GEP rolling horizon, then if banking is allowed, then it enables to reach 37.4% of the difference between the least favorable cost (with no intertemporal flexibility) and the optimum cost (banking and borrowing is allowed). Without banking : increasing of 1.04%. With banking : increase by 0.19%
- In our case, even facing limited foresight, a banking strategy enables to reach 82.2% of the carbon efficiency allowed in the perfect foresight case & full flexibility (compared with a no-flexible system). The investment trajectory and the emissions between perfect foresight case of annual and pluriannual policies. (cf. Figure 7)

Perfect carbon response

- When no frictions are added to the simulation module, the remaining differences between the optimization and the simulation module are due to indivisibilities. Indivisibilities derive from the gradient descent parameters. While GEP is solved with a continous solution, an investment loop as the one used in our simulation model is solved with a discrete path. In the litterature, indivisibities leads to underinvestments. In our model, because of the carbon price loop stop criterion, indivisibilites can lead to overinvestments and low carbon emissions.
- Theoretically, it is possible to solve the GEP as a Mixed Integer Problem (MIP). Such formulation has two important drawbacks which explain the choice of a continuous GEP. First solution of the MIP are not necessarily close to the optimal solution found in the continous problem. Because our model need a benchmark for the best solution and

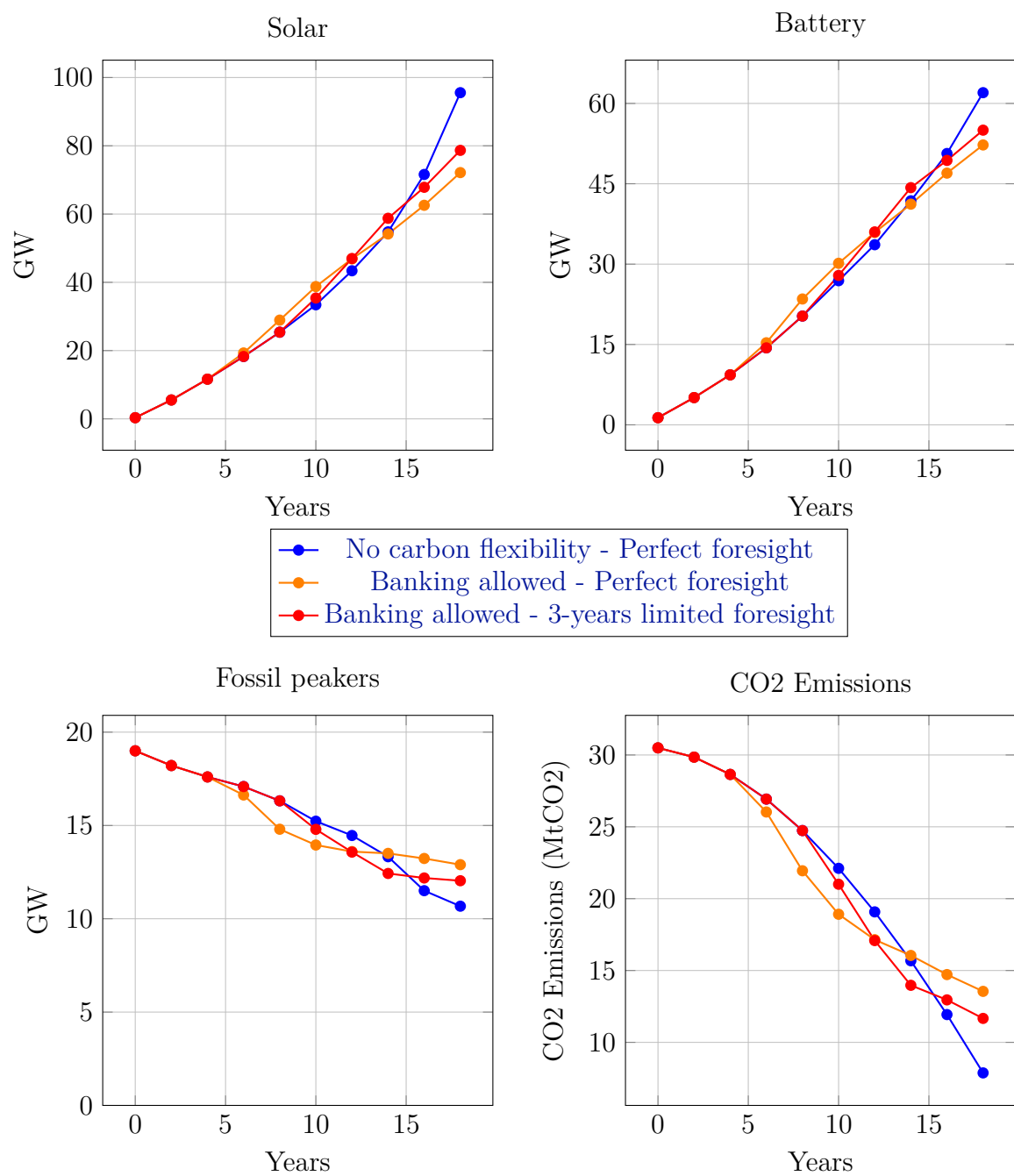


Figure 7: Capacities trajectories and CO₂ emissions in an optimization model with perfect and limited foresight

because there is no market reality in avoiding a continuous adaptation of the capacities, MIP is not used to compute the anticipation used in the simulation model. Second, using a MIP deprives us from using dual variables to build a co-determination of carbon and electricity price which is a key contribution of this paper.

- Yet, it is possible then to compare investment trajectories (see Figure 8) and carbon price evolution (see Figure 9). Investments in a simulation model with perfect foresight are close enough to compare with the system with frictions.
- The evolution of the carbon price in the case of a dynamic response of the carbon price leads to a trajectory of CO₂ emissions that respects the emission quotas without too strong a deviation from the optimal investments resulting from the optimization module.

Carbon price with frictions

- Adding a limited vision to the model enables us to study the resilience of a system that allows emissions banking in the presence of friction on the carbon market that could significantly limit its efficiency. It would be possible, when faced with limited foresight, for the system to use up its carbon allowances too quickly and lose efficiency to a system that cannot store carbon permits but has a perfect view of the carbon market.
- Using a banking property reduces the volatility of CO₂ prices (compared with prices derived from the optimization model). A system with banking is significantly less sensitive to limited foresight than a system that does not allow the storage of CO₂ emission permits with regard to the variability of the carbon price in relation to an optimal reference price (derived from the optimization model adapted to each regulation). See Figures 10 and 11 for details. For the performance of a system with banking to be below that of a system without flexible allocation of emission permits, it is necessary to increase its myopia by 3 to 5 years.
- The more the limited foresight is reduced in a system using CO₂ emissions banking, the lower the system costs. Figure 12 depicts the cost difference between the variety of studied scenarios. Even in the worst case (with only a two-year limited foresight), system allowing banking still have a lower cost than a system with no carbon permits allocation flexibility in the best case (with a perfect foresight).

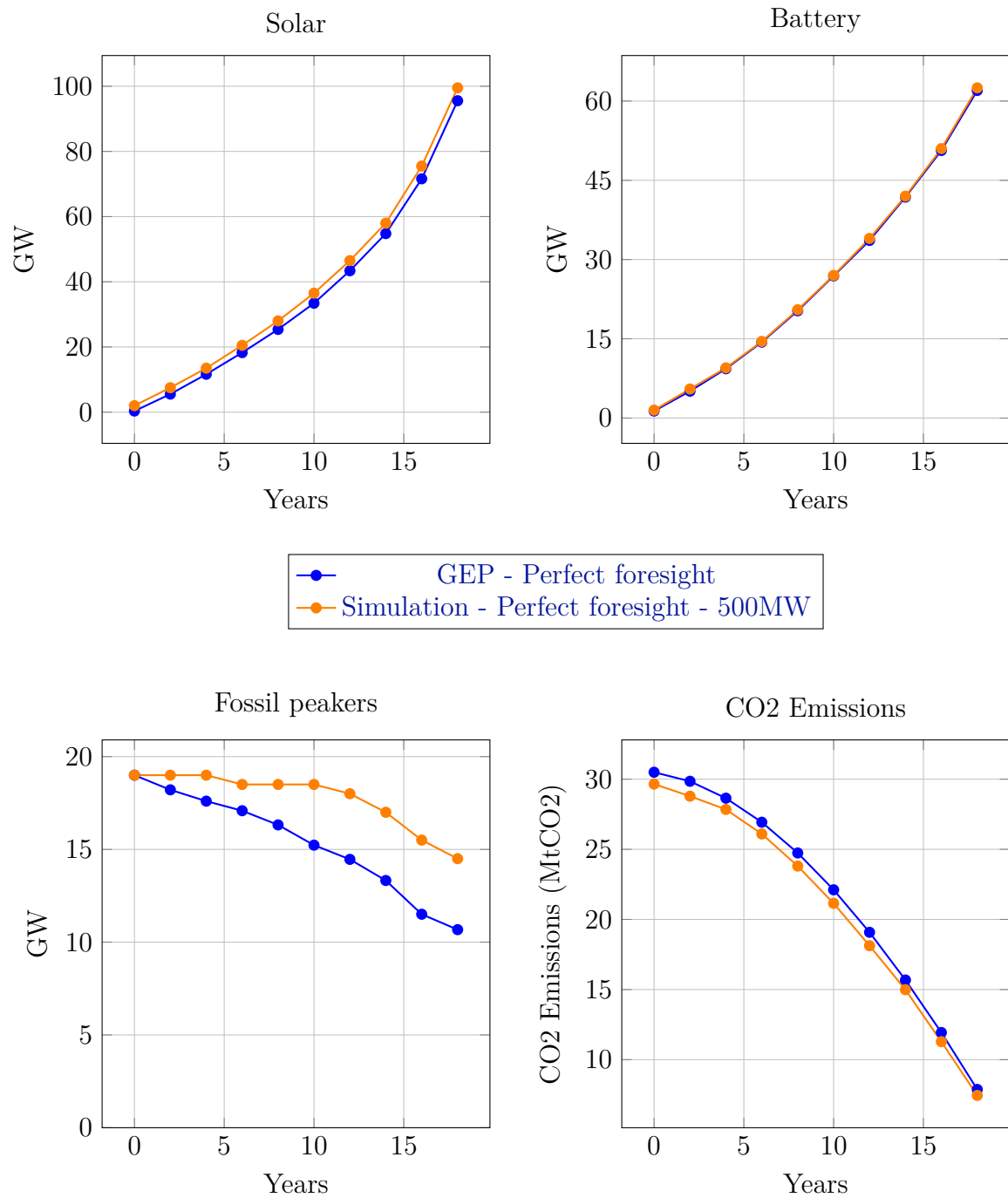


Figure 8: Capacities trajectories and CO₂ emissions in a simulation model with perfect foresight (with an annual CO₂ budget cap)

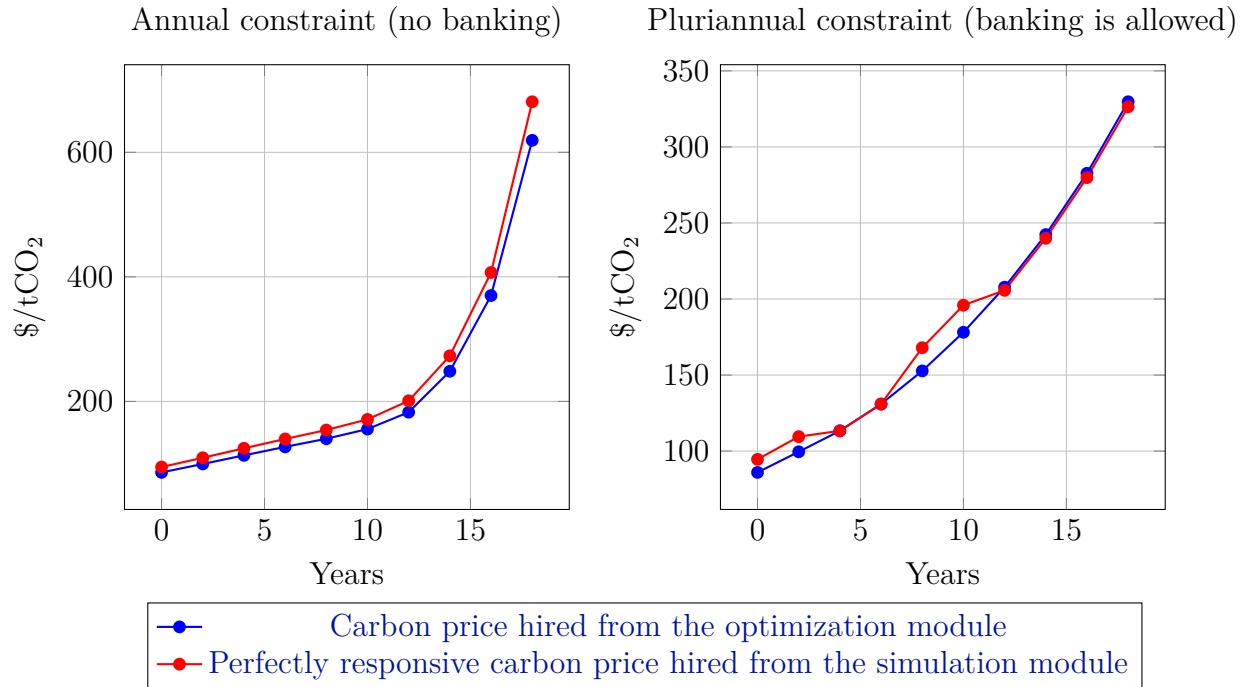


Figure 9: Carbon price evolution (in perfect foresight models)

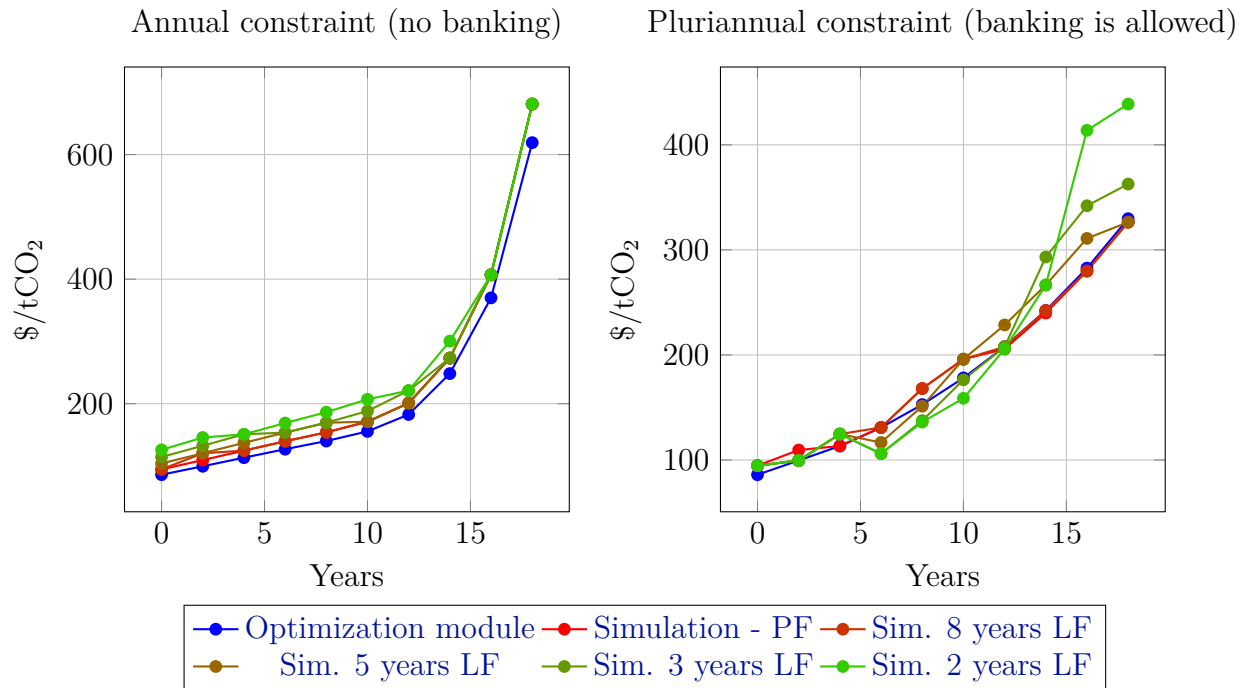


Figure 10: Carbon price evolution with perfect -PF- and limited foresight -LF-

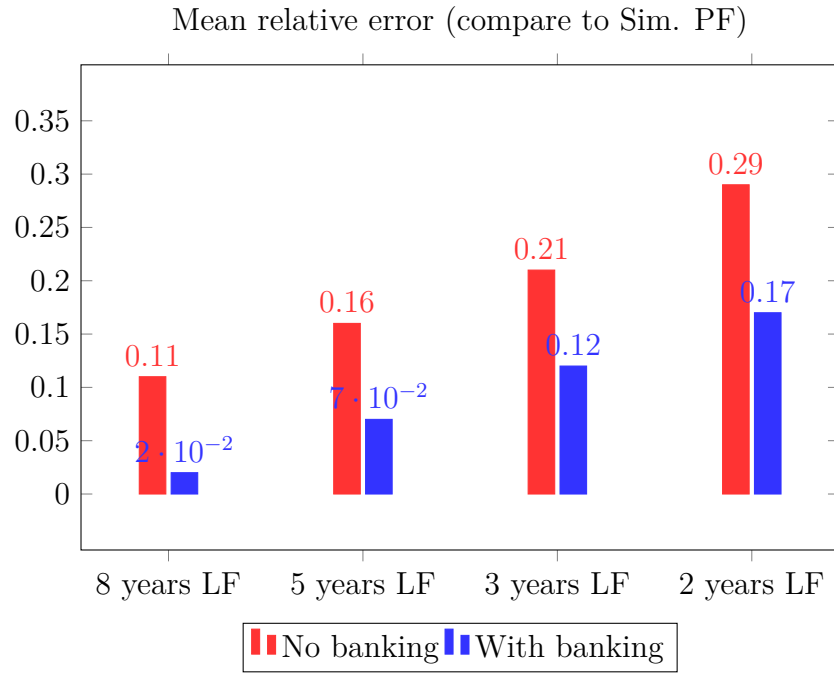


Figure 11: Carbon price deviation of carbon markets facing limited foresight

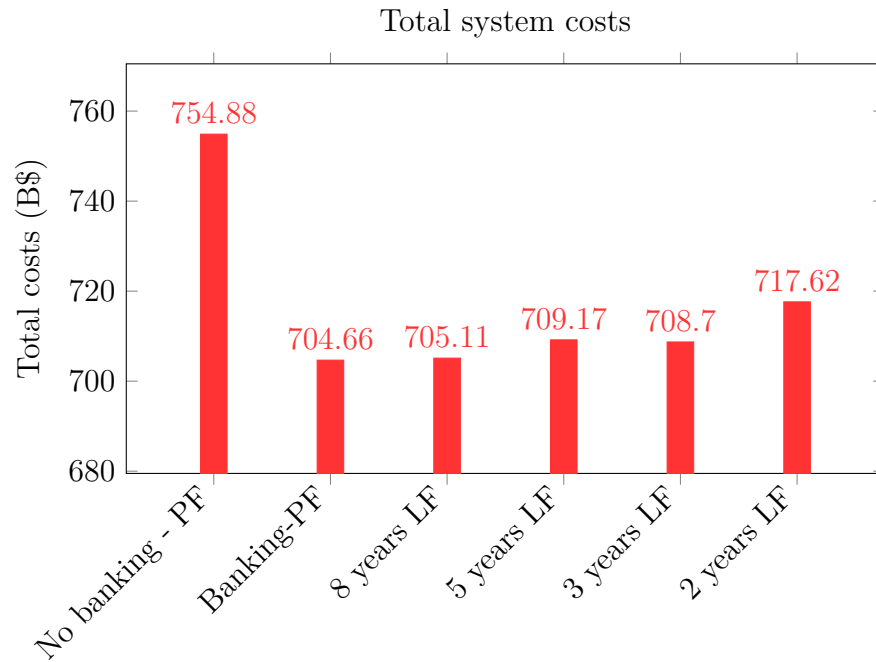


Figure 12: Total system costs facing limited foresight (in a system with banking)

6 Conclusion

Short summary & policy implications

- Co-determination of carbon and electricity prices allows to characterize the efficiency of a carbon market design in a system facing limited foresight. The development of this co-determination is possible thanks to a study of the behaviour of the dual variables of the optimization module used and thanks to a carbon price loop in the simulation model.
- The study of the dual variables leads to a new method to compute carbon price in an optimization allowing carbon permits banking.
- This carbon price determination is verified thanks to a study on the equivalence between a quantity-based policy and a price-based policy derived from the developed method. This equivalence is nevertheless lost when degeneracy occurs (e.g. when too many optimal solution coexist).
- Allowing banking in a system without market frictions allows to reach a lower system cost.
- In presence of limited foresight, a system allowing carbon permits banking is still better than a system without banking.

Future work

- The loss of the equivalence between a quantity-based and a price-based carbon policy when degeneracy occurs is still a preliminary result to the extent it has been only proven with a fictitious generating fleet and carbon budget path. A developed characterisation of the needed generating fleet is part of our future work.
- Our study develops an analysis of a carbon market design allowing carbon permits banking. The efficiency of this market design is tested with different carbon budget paths and different foresight (perfect and various limited time horizon). Other carbon market frictions will be studied (such as risk-aversion).
- Our study shows that in the presence of carbon market frictions the system does not reach the optimal investment trajectories. To fulfill the gap, the use of long-term contracts will be studied as a complementarity remuneration mechanism for decarbonized assets.

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A Notations and units

Table 1: Notations and units

Sets and indices	
\mathcal{H}	Set of hours in a year, indexed by h
\mathcal{Y}	Set of years, indexed by y
\mathcal{W}	Set of weather scenarios, indexed by w
\mathcal{G}	Set of conventional dispatch technologies, indexed by g
\mathcal{V}	Set of variable renewable energy technologies, indexed by v
\mathcal{S}	Set of storage technologies, indexed by s
\mathcal{T}	Set of all technologies ($\mathcal{T} = \mathcal{G} \cup \mathcal{V} \cup \mathcal{S}$), indexed by t
\mathcal{U}_t	Set of units of technology t , indexed by u
Parameters and variables	
β	Discount factor
Π_w	Probability of weather scenario w
$D_{y,w,h}$	Load in year y , weather scenario w and hour h [MW]
$\lambda_{y,w,h}$	Marginal cost of electricity in year y , weather scenario w and hour h [\$/MWh]
$OC_{y,t}$	Annual fixed O&M cost for technology t [\$/MW/yr]
$IC_{y,t}$	Investment cost annuity for technology t [\$/MW/yr]
$VC_{y,t}$	Annual operating cost for technology t [\$/MWh]
ζ_t	Carbon intensity of technology t [tCO ₂ /MWh]
Q_y	Annual CO ₂ emissions cap [tCO ₂]
$\pi_y^{CO_2}$	Carbon price in year y
ℓ_t	Lifespan of technology t [yr]
$n_{y,t}$	Number of operating units in year y for technology t
$n_{y,t}^+$	Number of developed units in year y for technology t
$n_{y,t}^-$	Number of closed units in year y for technology t
$\kappa_{y,t}$	Total installed capacity in year y for technology t [MW]
$\alpha_{t,w,h}$	Hourly availability of technology t [%]
k_t	Power capacity of technology t [MW/unit]
$q_{t,y,w,h}$	Production of technology t in year y , weather scenario w and hour h [MW]
$c_{s,y,w,h}$	Power charged into technology s in year y , weather scenario w and hour h [MW]
$soc_{s,y,w,h}$	State of charge of technology s in year y , weather scenario w and hour h [MWh]
ρ_s	Charging and discharging efficiency of technology s [%]
d_s	Storage duration for technology s [hours]
$f_{y,w,h}$	Lost load in year y , weather scenario w and hour h [MW]
$VoLL$	Value of Lost Load [\$/MWh]
η_t	Efficiency of technology t

B Optimization Problem

Write the generic problem (as in Appendix B in Lebeau et al. 2024)

$$\min_{n, n^+, n^-, q, f, c} \sum_{y \in \mathcal{Y}} \beta^y \left\{ \sum_{w \in \mathcal{W}} \Pi_w \sum_{h \in \mathcal{H}} \left[\sum_{t \in \mathcal{T}} VC_{y,t} \cdot q_{t,y,w,h} + VoLL \cdot f_{y,w,h} \right] + \sum_{t \in \mathcal{T}} \left[OC_{y,t} \cdot n_{y,t} + IC_{y,t} \cdot n_{y,t}^+ \cdot \sum_{i=0}^{\min(l_t, \#\mathcal{Y}-y)} \beta^i \right] \right\} \quad (14)$$

The problem minimises the total cost of the system by varying the asset additions and withdrawals for each year of the simulation. Three types of cost are taken into account in the minimization. The investment cost $IC_{y,t} \cdot n_{y,t}^+$ proportional to the added units in the electricity mix, the fixed operating cost $OC_{y,t} \cdot n_{y,t}$ independent of electricity production but proportional to the actual electricity mix and the variable cost $VC_{y,t} \cdot q_{t,y,w,h}$ proportional to the production of the electric mix. The actualisation factor β is equal to $\frac{1}{1+ActualisationRate}$.

This minimization problem is subject to constraints. The first set of constraints represents the hourly dispatch, that is $\forall y \in \mathcal{Y}, \forall w \in \mathcal{W}$,

$$\forall h \in \mathcal{H}, \sum_{t \in \mathcal{T}} q_{t,y,w,h} + f_{y,w,h} = D_{y,w,h} + \sum_{s \in \mathcal{S}} c_{s,y,w,h}. \quad (15)$$

$$\forall h \in \mathcal{H}, t \in \mathcal{T}, q_{t,y,w,h} \leq k_t a_{t,w,h} n_{y,t}. \quad (16)$$

$$\forall h \in \mathcal{H}, s \in \mathcal{S}, soc_{s,y,w,h} \leq k_s d_s n_{y,s}. \quad (17)$$

$$\begin{aligned} \forall h \in \mathcal{H}^*, s \in \mathcal{S}, soc_{s,y,w,h} &= soc_{s,y,w,h-1} \\ &+ \rho_s c_{s,y,w,h-1} \\ &- q_{s,y,w,h-1} \rho_s. \end{aligned} \quad (18)$$

where (15) imposes load balance, (16) imposes the upper limit on generation (for simplicity

dynamic generation constraints such as rampup rates are not represented), (17) imposes the upper limit on stored energy, and (18) reflects the storage dynamics.

The fleet dynamic is represented by the following constraints :

$$\forall y \in \mathcal{Y}^*, h \in \mathcal{H}, t \in \mathcal{T}, n_{y,t} = n_{y-1,t} + n_{y,t}^+ \quad (19)$$

$$\forall y \in \mathcal{Y}, t \in \mathcal{T}, \text{ if } y + \ell_t \leq \#\mathcal{Y} : \sum_{i=y}^{y+\ell_t} n_{i,t} \geq 1 \quad (20)$$

The GEP also needs to be completed if we want to ensure that the system is decarbonized. To achieve this, one can use constraints presented in (2) or in (3).

C Simplified GEP

In order to study the behaviour of the GEP, a simpler case is studied in what follows.

Hypothesis Compared with the GEP previously introduced, the following general simplifications have been made.

- No weather scenarios
- No discount factor
- No failures (borne by one of the variable costs studied)
- No storage
- No fixed operating costs
- Invested assets have a very long lifespan compared with that of the study.
- Expense factors are always assumed to be equal to 1
- Only investments

Formulation The objective function is given as follows :

$$\min_{n, n^+, q} \sum_{y \in \mathcal{Y}} \beta^y \left\{ \sum_{h \in \mathcal{H}} \left[\sum_{t \in \mathcal{T}} V C_{y,t} \cdot q_{y,h,t} \right] + \sum_{t \in \mathcal{T}} \left[k_t \cdot I C_{y,t} \cdot n_{y,t}^+ \cdot \sum_{i=0}^{\#\mathcal{Y}-y} \beta^i \right] \right\} \quad (21)$$

Subject to the following constraints: Demand constraint:

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \quad \sum_{t \in \mathcal{T}} q_{y,h,t} = D_{y,h} \quad (\lambda_{y,h}) \quad (22)$$

Capacity limit constraint:

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad q_{y,h,t} \leq k_t \cdot n_{y,t} \quad (\mu_{y,h,t}) \quad (23)$$

capacity continuity constraint:

$$\forall y \in \mathcal{Y}^*, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad n_{y,t} = n_{y-1,t} + n_{y,t}^+ \quad (\nu_{y,t}) \quad (24)$$

Non-negativity:

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad q_{y,h,t} \geq 0 \quad (\epsilon_{y,h,t}^q) \quad (25)$$

$$\forall y \in \mathcal{Y}, \forall t \in \mathcal{T}, \quad n_{y,t} \geq 0 \quad (\epsilon_{y,t}^n) \quad (26)$$

$$\forall y \in \mathcal{Y}, \forall t \in \mathcal{T}, \quad n_{y,t}^+ \geq 0 \quad (\epsilon_{y,t}^+) \quad (27)$$

Annual carbon budget constraint :

$$\forall y \in \mathcal{Y}, \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} q_{y,h,t} \cdot \zeta_t \leq Q_y \quad (\gamma_y) \quad (28)$$

Pluriannual carbon budget constraint :

$$\forall y \in \mathcal{Y}, \sum_{i=0}^y \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} q_{i,h,t} \cdot \zeta_t \leq \sum_{i=0}^y Q_i \quad (\gamma_y) \quad (29)$$

Lagrangian

$$\begin{aligned}
& \sum_{y \in \mathcal{Y}} \left\{ \sum_{h \in \mathcal{H}} \left[\sum_{t \in \mathcal{T}} \beta^y \cdot VC_{y,t} \cdot q_{t,y,w,h} \right] + \sum_{t \in \mathcal{T}} \left[\beta^y \cdot k_t \cdot IC_{y,t} \cdot n_{y,t}^+ \cdot \sum_{i=0}^{\#\mathcal{Y}-y} \beta^i \right] \right. \\
& \quad + \sum_{h \in \mathcal{H}} \left[\lambda_{y,h} \cdot (D_{y,h} - \sum_{t \in \mathcal{T}} q_{y,h,t}) \right. \\
& \quad - \sum_{t \in \mathcal{T}} \mu_{y,h,t} \cdot (q_{y,h,t} - k_t \cdot n_{y,t}) \\
& \quad + \sum_{t \in \mathcal{T}} \nu_{y,t} \cdot (n_{y-1,t} + n_{y,t}^+ - n_{y,t}) \\
& \quad \left. \left. + \epsilon_{y,h,t}^q \cdot q_{y,h,t} + \epsilon_{y,t}^n \cdot n_{y,t} + \epsilon_{y,t}^+ \cdot n_{y,t} \right] \right. \\
& \quad \left. - \gamma_y \cdot \left(\sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} q_{y,h,t} \cdot \zeta_t - Q_y \right) \right\} \\
& \hspace{15em} (30)
\end{aligned}$$

KKT With an annual carbon constraint (banking is forbidden)

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad \beta^y VC_{y,t} - \lambda_{y,h} - \mu_{y,h,t} + \epsilon_{y,h,t}^q - \zeta_t \cdot \gamma_y = 0 \quad (31)$$

With a pluriannual carbon constraint (banking is allowed)

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad \beta^y VC_{y,t} - \lambda_{y,h} - \mu_{y,h,t} + \epsilon_{y,h,t}^q - \zeta_t \cdot \sum_{j=y}^{\#\mathcal{Y}} \gamma_j = 0 \quad (32)$$

$$\forall y \in \mathcal{Y}, \forall t \in \mathcal{T}, \quad \beta^y \cdot k_t \cdot IC_{y,t} \cdot \sum_{i=y}^{\#\mathcal{Y}} \beta^i + \nu_{y,t} + \epsilon_{y,t}^+ = 0 \quad (33)$$

$$\forall y \in \mathcal{Y} \setminus \{\#\mathcal{Y}\}, \forall t \in \mathcal{T} \quad \sum_{h \in \mathcal{H}} k_t \cdot \mu_{y,h,t} - \nu_{y,t} + \nu_{y+1,t} + \epsilon_{y,t}^n = 0 \quad (34)$$

$$y = \{\#\mathcal{Y}\}, \forall t \in \mathcal{T} \quad \sum_{h \in \mathcal{H}} k_t \cdot \mu_{y,h,t} - \nu_{y,t} + \epsilon_{y,t}^n = 0 \quad (35)$$

Complementarity conditions:

$$D_{y,h} - \sum_{t \in \mathcal{T}} q_{y,h,t} = 0 \quad (36)$$

$$\begin{cases} k_t \cdot n_{y,t} - q_{y,h,t} = 0 \\ \mu_{y,h,t} = 0 \end{cases} \quad (37)$$

$$n_{y-1,t} + n_{y,t}^+ - n_{y,t} = 0 \quad (38)$$

$$\begin{cases} q_{y,h,t} = 0 \\ \epsilon_{y,h,t}^q = 0 \end{cases} \quad (39)$$

$$\begin{cases} n_{y,t} = 0 \\ \epsilon_{y,t}^n = 0 \end{cases} \quad (40)$$

$$\begin{cases} n_{y,t}^+ = 0 \\ \epsilon_{y,t}^+ = 0 \end{cases} \quad (41)$$

With an annual carbon constraint (if banking is forbidden)

$$\begin{cases} Q_y - \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} q_{y,h,t} \cdot \zeta_t = 0 \\ \gamma_y = 0 \end{cases} \quad (42)$$

With a pluriannual carbon constraint (if banking is allowed)

$$\begin{cases} \sum_{i=0}^y Q_i - \sum_{i=0}^y \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} q_{i,h,t} \cdot \zeta_t = 0 \\ \gamma_y = 0 \end{cases} \quad (43)$$

C.1 Carbon price and dual variables

The aim is to express the marginal cost of a time step (y, h) and to demonstrate the cost recovery of a technology t invested in year y . On se concentre sur le cas où la contrainte carbone est annuelle.

Marginal cost

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad \beta^y V C_{y,t} - \lambda_{y,h} - \mu_{y,h,t} + \epsilon_{y,h,t}^q - \zeta_t \cdot \gamma_y = 0 \quad (44)$$

Let y and h , let t be the marginal technology (it must exist), then the KKT conditions give: $\mu_{y,h,t} = 0$ (because it is not inframarginal) and $\epsilon_{y,h,t}^q = 0$. This gives :

$$\lambda_{y,h} = \beta^y V C_{y,t} - \zeta_t \cdot \gamma_y \quad (45)$$

The equation gives the expression of marginal cost as a function of variable cost and the dual variable of the carbon constraint.

Carbon price The canonic form of the associated dual variable expression gives (according to our formulation) :

$$\forall y \in \mathcal{Y}, \quad \gamma_y \leq 0 \quad (46)$$

We want to set a price for CO_2 so that

$$\lambda_{y,h} = V C_{y,t} = \frac{C_{y,t}}{\rho_t} + \xi_t \cdot \pi_y^{CO_2} \quad (47)$$

because we want to be equivalent both cost minimization problems with and without CO_2 budget constraint. It follows quite naturally that :

$$\pi_y^{CO_2} = -\beta^{-y} \gamma_y \quad (48)$$

The definition of the CO_2 price is similarly justified in the case of writing the problem with a multi-year carbon constraint.

C.2 Equivalence

When no banking occurs To check the validity of the introduction of the formula interpreting a sliding sum of dual variables as a carbon price in the case where the system allows banking, it is necessary to check if both definition of the carbon price coincide if the system which allows banking decides to not bank emissions permits (because of demand data or other parameters influence - see 5.3). We write below a extraction of both optimization problem. By writing the KKTs of the two optimization problems, we have the following two elements (we note with a superscript a the annual case and with a superscript p the case where the carbon constraint is pluriannual).

With an annual carbon constraint:

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad \beta^y V C_{y,t} - \lambda_{y,h}^a + \mu_{y,h,t}^a - \epsilon_{y,h,t}^{qa} + \zeta_t \cdot \gamma_y^a = 0 \quad (49)$$

With a pluriannual carbon constraint:

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad \beta^y V C_{y,t} - \lambda_{y,h}^p + \mu_{y,h,t}^p - \epsilon_{y,h,t}^{qp} + \zeta_t \cdot \sum_{j=y}^{\#\mathcal{Y}} \gamma_j^p = 0 \quad (50)$$

When the system does not store CO_2 permits, the feasible solutions for the two optimization problems are identical. This is because the carbon constraint is propagated from one level to the next, resulting in a problem with an annual carbon constraint. In this case, the dual variables associated with the identical constraints between the two problems are equal. We therefore have :

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \quad \lambda_{y,h}^a = \lambda_{y,h}^p \quad (51)$$

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad \mu_{y,h,t}^a = \mu_{y,h,t}^p \quad (52)$$

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad \epsilon_{y,h,t}^{qa} = \epsilon_{y,h,t}^{qp} \quad (53)$$

In these cases, it can be written :

$$\forall y \in \mathcal{Y}, \quad \gamma_y^a = \sum_{j=y}^{\#\mathcal{Y}} \gamma_j^p \quad (54)$$

If banking occurs - Hoteling's rule Precedent paragraphs give:

$$\pi_y^{CO_2} = -\beta^{-y} \sum_{j=y}^{\#\mathcal{Y}} \gamma_j^p \quad (55)$$

We also know that when the system stores CO_2 permits in a year y , then $\gamma_y^p = 0$ (because the carbon constraint of the year y is thus not active).

Let's consider a permit banking period running from year y_1 to year y_2 (i.e. the system stores its permits up to the year y_2 excluded, when it can no longer store (this year necessarily exists, as it is maximized by the year at the end of the horizon). Then

$$\pi_{y_1}^{CO_2} = -\beta^{-y_1} \sum_{j=y_1}^{\#\mathcal{Y}} \gamma_j^p \quad (56)$$

Then

$$\forall j \in [y_1, y_2 - 1], \gamma_j^p = 0 \quad (57)$$

Then

$$\pi_{y_1}^{CO_2} = -\beta^{-y_1+y_2} \beta^{-y_2} \sum_{j=y_2}^{\#\mathcal{Y}} \gamma_j^p \quad (58)$$

Then

$$\pi_{y_2}^{CO_2} = \beta^{y_1-y_2} \pi_{y_1}^{CO_2} = (1 + ActualisationRate)^{y_2-y_1} \pi_{y_1}^{CO_2} \quad (59)$$

Hoteling's rule is verified.

Particularity - switch price At the start of the reasoning, we choose the marginal t_1 technology. We then continue the reasoning by choosing another technology t_2 . Equalizing with $\lambda_{y,h}$, then :

$$\beta^y \cdot VC_{y,t_1} + \xi_{t_1} \gamma_y = \beta^y \cdot VC_{y,t_2} + \mu_{y,h,t_2} + \xi_{t_2} \cdot \gamma_y \quad (60)$$

Then

$$\gamma_y = \frac{\beta^y \cdot (VC_{y,t_2} - VC_{y,t_1}) + \mu_{y,h,t_2}}{\xi_{t_1} - \xi_{t_2}} \quad (61)$$

In particular, if the parameters finally impose that this second technology is marginal, then $\mu_{y,h,t_2} = 0$ and we find the carbon fuel switch equation.

C.3 Cost recovery

KKT give us:

$$\forall y \in \mathcal{Y}, \forall t \in \mathcal{T}, \quad k_t \cdot IC_{y,t} \cdot \sum_{i=y}^{\#\mathcal{Y}} \beta^i + \nu_{y,t} + \epsilon_{y,t}^+ = 0 \quad (62)$$

Given that we are looking to see if the technology invested recovers its costs, we have : $\epsilon_{y,t}^+ = 0$ et $\epsilon_{y,t}^n = 0$.

$$\forall y \in \mathcal{Y}, \forall t \in \mathcal{T}, \quad k_t \cdot IC_{y,t} \cdot \sum_{i=y}^{\#\mathcal{Y}} \beta^i = -\nu_{y,t} \quad (63)$$

Let y be $\in [0, \#\mathcal{Y} - 1]$, then

$$\nu_{y+1,t} - \nu_{y,t} = - \sum_{h \in \mathcal{H}} k_t \mu_{y,h,t} \quad (64)$$

With a sum for i in range 0 to $y-1$, then

$$\nu_{y,t} - \nu_{0,t} = - \sum_{i=0}^{y-1} \sum_{h \in \mathcal{H}} k_t \mu_{i,h,t} \quad (65)$$

Then

$$\nu_{y,t} = \nu_{0,t} - \sum_{i=0}^{y-1} \sum_{h \in \mathcal{H}} k_t \mu_{i,h,t} \quad (66)$$

In another hand we have (because $\epsilon_{\# \mathcal{Y},t}^n = 0$):

$$\nu_{\# \mathcal{Y},t} = \sum_{h \in \mathcal{H}} k_t \mu_{\# \mathcal{Y},h,t} \quad (67)$$

KKT give us :

$$\sum_{h \in \mathcal{H}} k_t \mu_{\# \mathcal{Y}-1,h,t} - \nu_{\# \mathcal{Y}-1,t} + \nu_{\# \mathcal{Y},t} = 0 \quad (68)$$

Then

$$\sum_{h \in \mathcal{H}} k_t \mu_{\# \mathcal{Y}-1,h,t} + \sum_{h \in \mathcal{H}} k_t \mu_{\# \mathcal{Y},h,t} = \nu_{0,t} - \sum_{i=0}^{\# \mathcal{Y}-2} \sum_{h \in \mathcal{H}} k_t \mu_{i,h,t} \quad (69)$$

Then

$$\nu_{0,t} = k_t \sum_{i=0}^{\# \mathcal{Y}} \sum_{h \in \mathcal{H}} \mu_{i,h,t} \quad (70)$$

Then

$$\nu_{y,t} = k_t \sum_{i=y}^{\# \mathcal{Y}} \sum_{h \in \mathcal{H}} \mu_{i,h,t} \quad (71)$$

Then

$$\forall y \in \mathcal{Y}, \forall t \in \mathcal{T}, \quad k_t \cdot IC_{y,t} \cdot \sum_{i=y}^{\# \mathcal{Y}} \beta^i = - \sum_{i=y}^{\# \mathcal{Y}} \sum_{h \in \mathcal{H}} k_t \cdot \mu_{i,h,t} \quad (72)$$

Let's write $\mu_{y,h,t}$

$$\forall y \in \mathcal{Y}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad -\mu_{y,h,t} = -\beta^y V C_{y,t} + \lambda_{y,h} - \epsilon_{y,h,t}^q + \zeta_t \cdot \gamma_y \quad (73)$$

We study the sum of (72), distinguishing whether the power plant is marginal or inframarginal.

If the power plant is inframarginal ($0 < q_{y,h,t} < k_t$), then $\mu_{y,h,t} = 0$ and $\epsilon_{y,h,t} = 0$. Then k_t can be written as $q_{y,h,t}$ in (72) (because the term inside the sum is then zero). One can also replace $\mu_{i,h,t}$ by its expression : $-\mu_{y,h,t} = -\beta^y V C_{y,t} + \lambda_{y,h} + \zeta_t \cdot \gamma_y$

If the power plant is marginal ($q_{y,h,t} = k_t$), then $\mu_{y,h,t} \neq 0$ and $\epsilon_{y,h,t} = 0$. One can replace k_t by $q_{i,h,t}$ in (72) because the power plant is marginal. One can also replace $\mu_{i,h,t}$ by its expression : $-\mu_{y,h,t} = -\beta^y V C_{y,t} + \lambda_{y,h} + \zeta_t \cdot \gamma_y$

If the power plant is not called ($q_{y,h,t} = 0$), then $\mu_{y,h,t} = 0$. Then k_t can be written as $q_{y,h,t}$ in (72) (because the product is equal to zero). One can also replace $\mu_{i,h,t}$ by its expression : $-\mu_{y,h,t} = -\beta^y V C_{y,t} + \lambda_{y,h} + \zeta_t \cdot \gamma_y$ (even if this expression does not correspond to $\mu_{i,h,t}$, it is possible to do this because it is multiplied by $q_{y,h,t} = 0$).

Then it comes the cost recovery equation:

$$\forall y \in \mathcal{Y}, \forall t \in \mathcal{T}, \quad k_t \cdot I C_{y,t} \cdot \sum_{i=y}^{\#\mathcal{Y}} \beta^i = \sum_{i=y}^{\#\mathcal{Y}} \sum_{h \in \mathcal{H}} q_{i,h,t} \cdot (\lambda_{i,h} - \beta^y V C_{i,t} + \zeta_t \cdot \gamma_i) \quad (74)$$

This allows us to observe that the plant built necessarily recovers its costs thanks to all the hourly time steps where it is not marginal. In the case where the carbon constraint is pluriannual, similar reasoning allows us to obtain that :

$$\forall y \in \mathcal{Y}, \forall t \in \mathcal{T}, \quad k_t \cdot I C_{y,t} \cdot \sum_{i=y}^{\#\mathcal{Y}} \beta^i = \sum_{i=y}^{\#\mathcal{Y}} \sum_{h \in \mathcal{H}} q_{i,h,t} \cdot (\lambda_{i,h} - \beta^y V C_{i,t} + \zeta_t \cdot \sum_{j=y}^{\#\mathcal{Y}} \gamma_j) \quad (75)$$