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«Derisking Electricity Prices For Decarbonisation: A Novel Perspective on Market Incompleteness Through Irreversibility»

by

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Derisking Electricity Prices For Decarbonisation: A novel perspective on market incompleteness through irreversibility

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Abstract

Long-term electricity contracting has remained an unresolved issue in European markets, with growing implications for decarbonising industries. This 'missing market' failure indeed let investors exposed to the long-term electricity market volatility, increasing the cost of capital, and hence reducing investments in both new generation and electrification of usages. In this paper, we argue that infrastructure-heavy industrial decarbonisation investments stand apart due to their unique characteristics: they are subject to strong ambiguity - rather than risk -, and are irreversible by design. Building on these insights, we develop a bilateral contracting model that accounts for ambiguity aversion and investment irreversibility, tailored to the dynamics between energy producers and industrial consumers. Using the real-world case of ArcelorMittal and EDF, we demonstrate that the current market design prevents parties from reaching a mutually advantageous agreement. Finally, we discuss implications for European firms and policy measures to overcome these barriers and stimulate low-carbon investment across both sectors.

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1 Introduction

The European Union's achievement of its net-zero targets relies on the decarbonisation of the energy and industrial sectors, which respectively account for 27.41% and 20.26% of its greenhouse gas emissions European Environment Agency [2024]. Since industrial decarbonisation is dependent on electrification routes, effectively replacing fossil fuels in their production processes, both sector are mutually dependent and face symmetric risks. Moreover, their respective transitions require new infrastructure-heavy technological adoption and access to low financing costs. Given these dynamics, one would expect the electricity futures market to facilitate risk hedging: but while short-term hedging positions are available, long-term financial market remains incomplete - or even non existent [Lucy and Kern, 2021, Billimoria et al., 2024].

Without mechanisms to hedge price risk, both sectors face significant uncertainty, endangering investment in low-carbon technologies. This challenge is not merely about energy: it concerns Europe's industrial future, economic resilience, and strategic sovereignty, especially in the context of the Russian invasion of Ukraine and geopolitical instability under a Trump administration. Recent policy efforts, such as the EU's Clean Industrial Deal, displayed growing awareness of this issue. Long-term bilateral contracting tools like Power Purchase Agreements (PPAs) have been increasingly discussed as solutions to stabilise electricity prices and secure investment while limiting government spendings. However, for now, policymakers have mostly focused on administrative barriers to PPA signings Draghi [2024]. As a result, PPA adoption has remained low, compared to what traditional economic wisdom on risk aversion might have suggested. In this paper, we aim to unravel the implications of this missing financial market on both sectors' contracting and investment decision, and explain why negotiations the number of contracts involving heavy industries (such as the steel sector) remains low.

It is important to note that electricity price volatility is inherently high due to the need for continuous supply-demand balance [Bessembinder and Lemmon, 2002, Božić et al., 2020]. Moreover, growing renewable penetration brings both downward price pressure [Kolb et al., 2020, Cevik and Ninomiya, 2022], new risks such as the cannibalization effect [Halttunen et al., 2020], and have an ambiguous effect on volatility Cevik and Ninomiya [2022]. However, conflicting interests between power producers and consumers, along with market frictions, have limited the adoption of electricity futures contracts. Producers seek long-term price stability to secure financing, while off-takers often perceive these contracts as liabilities during periods of declining market prices [Simshauser, 2019, Lucy and Kern, 2021]. Additionally, the volatility of electricity markets, driven by supply-demand imbalances and geopolitical shocks, exacerbates these challenges [European Central Bank., 2021]. We are then confronted to a missing-market problem, extensively studied in the electricity markets literature (i.e. [Newbery, 2016], [Abada et al., 2019], [Simshauser, 2019]). [Mays et al., 2022] emphasizes the severe consequences of this incompleteness, including the risk of private agents facing bankruptcy after extreme events. Understanding why these markets fail to develop is crucial for aligning industrial decarbonization with secure electricity pricing.On top of these limitations, market incompleteness (as exposed in Magill and Quinzii [1996]) entails a lack of foresight and market readability, pushing companies to make subjective anticipations on the future of electricity prices and leading to a situation of *ambiguity* Thijssen [2011]. Ambiguity differs from risk as it arises from incomplete knowledge about future probabilities of outcomes.

In 2023, the French government demonstrated awareness of these issues by promoting a PPA between its leading steel manufacturer ArcelorMittal and historical power producer EDF, looking to secure their respective investments by leveraging their complementarity. However, they could not reach an agreement: ArcelorMittal postponed its decarbonization project, citing electricity market prices and uncertainty as major barriers to investment. In this context, policymakers must assess whether private actors can adequately hedge risks through bilateral agreements like PPAs or if new market regulations such as state-backed Contracts for Difference (CfDs), market maker obligations, or broader market reforms, are necessary to close this gap. We look to provide concrete answers to this question.

This paper contributes to the literature by correcting for two characteristics that have usually been overlooked in the discussion around PPA signings for industrials: investment irreversibility and exposure to ambiguity on top of risk. While traditional models typically assume investment reversibility, in reality, industrial assets are highly capital-intensive and cannot be liquidated at will - leading to irreversibility premiums Dixit and Pindyck [1994]. Moreover, ambiguity in policy-driven markets stemming from regulatory shifts and geopolitical events push agents towards cautious investment decisions and create barriers to these signings. Building on Thijssen [2011], we model market incompleteness as a source of ambiguity and analyse its impact on investment decisions. We start by introducing an augmented real options model where firms choose between investing independently, securing prices via bilateral contracts - such as PPA in the electricity sector -, or delaying investment. We then extend this model to focus on the bilateral contracting framework, capturing how electricity producers and industrial consumers navigate market incompleteness.

Finally, we apply this model to study the ongoing negotiations between ArcelorMittal and EDF by highlighting that decarbonisation presents a profound challenge for policymakers and private agents, characterized not only by traditional risk but also *ambiguity*. Traditional approaches to risk aversion, such as those rooted in von Neumann and Morgenstern [1944] and Markowitz [1952], often fail to capture the full scope of uncertainty in such markets. In the context of electricity and carbon markets, ambiguity seems especially pertinent. These markets' fundamentals indeed depend on systemic uncertainties such as future climate damages, political decisions, or technological change (Millner et al. [2013]). Their probabilities hence become difficult, if not impossible to estimate - even if there were perfect information sharing across the whole economy.

A rather restricted strand of literature looks to unravel the reasons for electricity market incompleteness. By applying our model to the case of PPA signings, we also look to contribute to the literature on incomplete electricity market by bringing new arguments to the table. Among them, [Abada et al., 2019] study this phenomenon through a modelling approach, and find that focusing on short-term efficiency prevents markets from being efficient in the long-term. [Batlle et al., 2023] attributes it to the lack of demand-side interest in hedging. [Schittekatte and Batlle, 2023] attributes it to barriers to entry, and hints that the introduction of market making obligations in electricity markets may resolve the issue. Most recently, [Billimoria et al., 2024] used a model inspired from the insurance literature to show that when leaving theoretical models and incorporating real-world constraints such as financing, the high volatility of prices during extreme events explains the lack of contracting. In this paper, we contribute to the debate by using model-based case studies to explain the lack of contract signings between consumers and producers.

Our findings demonstrate that private PPAs, without additional policy, will not be able to provide long-term stability required for large-scale industrial electrification. In the case of heavy industrials, even if states were to subsidise industrials and provide clarity on future carbon prices, electricity market uncertainty would outweigh these public efforts and prevent PPA signings. To simultaneously support new generation capacity, such as nuclear energy, and industrial decarbonization, more targeted policy interventions are necessary. If policymakers aim to facilitate both new power generation and large-scale industrial electrification, they must recognize the complexity of this challenge and design mechanisms beyond existing PPA frameworks. Addressing market incompleteness at its root is essential for securing industrial competitiveness, ensuring policy coherence, and reinforcing economic resilience in an increasingly uncertain world.

2 The Model - General Setting

In this section, we will introduce our reader to our bilateral contracting model in a generalised fashion, so that this model may be applied to other markets and used in other fields or research. The first three subsections will set the mathematical background for our model. The following will focus on economic analysis and comments on the various effects at play.

2.1 Assumptions

This section provides an overview of all assumptions concerning financial market prices, the profits of each agent, and their risk aversion. We consider two rational investors willing to enter a market : one on the supply side (hereafter called the *producer* p); and one on the demand side (hereafter called the *consumer* c) .They are endowed with an initial capital, which has to be invested at in :

1. Financial assets : a risk-free bond R and a market portfolio S ;
2. A real asset allowing the project carrier to:
   (a) Sell a good at price P - exclusively available for the producer p
   (b) Buy a good at price P - exclusively available for the consumer c;
3. The same real asset, but hedged by a Bilateral Contract (BC)

2.1.1 Financial assets

First of all, investors can first invest their money in a risk-free asset R (i.e. sovereign bond) with a fixed return r, and/or in a market portfolio S. We then assume that market portfolio follows a Geometric Brownian Motion (GBM) process, with drift parameter μ_S and volatility σ_S.

$$\frac{dR}{R} = rdt \tag{1}$$

$$\frac{dS}{S} = \mu_S dt + \sigma_S dB_t \tag{2}$$

These two financial assets represent all other investment opportunities available to the investors, if they decide not to invest in their real assets. We consider these markets to be financially complete, so that the market risk B_t can be fully priced with non-arbitrage methods.

2.1.2 Real asset

Instead of investing their capital on the financial market, the two investors can also buy a real asset with price P, representing the underlying value determinant for each asset. Similarly to the market portfolio S, we model the real asset's price P as a GBM, with a drift μ_P, a volatility σ_P:

$$\frac{dP}{P} = \mu_P dt + \sigma_P dW_t$$

Following [Henderson, 2007], we consider that the two driving Brownian motions are correlated with $\rho \in [-1 : 1]$. Hence, we can write that $dW_t = \rho dB_t + \sqrt{1-\rho^2} dZ_t$ with Z_t a third Brownian Motion, independent on B_t. By trading the risky asset S, the private investor can hedge part of the risk. However, the aforementioned idiosyncratic risk, captured by Z_t, remains. As both agents are risk-averse, they will penalize this remaining risk, and ask for a risk premium - which will be quantified in section 2.3.

2.1.3 Consumer

Let $U > 0$ be the initial endowment of the consumer, representing the benefits captured by the agent through its project, such as cost reductions and government subsidies, net of investment costs.

If $U > 0$[1], the investment is justified from the private agent's point of view, and we can write the *stochastic* profit π_t^c as :

$$\pi_t^c(P_t) = U - \int_t^{t+T} \lambda_\tau P_\tau d\tau$$

Where P_τ represents the real asset price at time τ, and λ_τ the discount factor. The notation $\pi_t^c(P_t)$ emphasises that the distribution of the stochastic profits depends, among other parameters, on the known asset price P_t at time t. Note that, in this model, we isolate the problem around P by assuming that all other markets are complete, and agents have the ability to hedge and foresight on other markets.

2.1.4 Producer

The producer is faced with a net, irreversible investment cost I, representing the initial capital expenditures that are needed to build the power plant and make it available [2]. Once the project is up and running, the

The producer will earn a yearly revenue from one unit of production, sold at market price P_τ. We can then write the producer's *stochastic* profit as:

$$\pi_t^p(P_t) = -I + \int_t^{t+T} \lambda_\tau P_\tau \, d\tau$$

For both the producer and the consumer, one can notice that the problem is time-invariant. Given the stationary stochastic processes followed by the market portfolio and the real asset's price, the distribution of the stochastic NPV depends only on the initial price P_t at the time of investment, but not on the investment time t itself. We will therefore write $\pi_t^i(P_t) = \pi^i(P_t)$ hereafter, for $i \in \{p, c\}$.

2.2 Hedging with a Bilateral Contract

For both agents, entering into a Bilateral Contract (BC) is mathematically equivalent to purchasing a series of futures or forwards that hedges against market price volatility. For the producer (resp. consumer), hedging against downward (resp. upward) price movements, this contract is similar to a short (resp. long) position on financial markets. The asset price is fixed, hence their revenues (resp. costs) remain stagnant over the period of the contract - all other things equal.

Signing a BC grants agents the possibility to stabilise their profits, and hedge price risk. Indeed, by signing a BC at a guaranteed strike price P_{strike} instead of the stochastic market price P_τ, their stochastic cumulative NPVs become independent on the idiosyncratic risk Z_τ :

$$\pi_{BC}^c(P_t) = U - \int_t^{t+T} \lambda_\tau P_{strike} d\tau$$

[1] Note that if $U < 0$, the investment never takes place as the investor is always losing.

[2] In the presence of variable costs, they can easily be incorporated as the contract is made over the yearly delivery of a fixed quantity of real asset

$$\pi^p_{BC}(P_t) = -I + \int_t^{t+T} \lambda_\tau P_{strike}\, d\tau$$

With T the length of the contract. We assume that there are no transaction costs [3].

For the *contracting case*, one can directly compute the expectancy under this natural probability measure, as all future cashflows are deterministic and equal to P_{strike}. Under the natural probability measure, B_τ and Z_τ are uncorrelated Brownian motions, so that the expectancy of the stochastic discount factor λ_τ is $\mathbb{E}^{\mathcal{P}}_t(\lambda_\tau) = e^{-r(\tau-t)}$. Thus, we can write:

$$NPV^p_{BC} = -I + P_{strike}\int_t^{t+T} \mathbb{E}^{\mathcal{P}}_t(\lambda_\tau)\, d\tau = -I + X \tag{3}$$

$$NPV^c_{BC} = U - P_{strike}\int_t^{t+T} \mathbb{E}^{\mathcal{P}}_t(\lambda_\tau)\, d\tau = U - X \tag{4}$$

$$\tag{5}$$

Where we introduce a new variable, the *contract price* X, which is the NPV of all contract's payments, identical to both agents :

$$X = K(r)P_{strike} \tag{6}$$

$$K(x) = \frac{1-e^{-xT}}{x} \tag{7}$$

$K(x)$ is an operator which, multiplied by a constant cash-flow, gives the NPV of the corresponding annuity on a period T, discounted at the rate x.

2.3 Introducing Ambiguity

Ambiguity aversion provides a more realistic lens for decision-making, as it accounts for the evolving and incomplete nature of market information [Chen and Epstein, 2002, Ilut and Schneider, 2022]. It was mathematically formalized through the work of Gilboa and Schmeidler [1989] and Bewley [2002], following foundational contributions by Ellsberg [1961]. Unlike risk, where uncertainty can be encapsulated within a single probability measure, ambiguity cannot. Consequently, uncertainty is represented by a set of probability measures, defined through the concept of *Knightian Uncertainty* or *k-ignorance*. This helps capture the uncertainty about the severity, timing, and effects of future events.

[3] This goes back to a situation where both parties have already found each other, or where a marketplace for contractualisation exists.

Following the steps of [Thijssen, 2011], we assume that the investor takes decisions based on unknown shareholder preferences, hence basing it on a stochastic discount factor λ_t. The discount factor then takes the form of an Ito diffusion:

$$\frac{d\lambda_t}{\lambda_t} = -\mu_\lambda dt - \zeta_B dB - \zeta_Z dZ$$

With $\lambda_0 = 1$, and ζ_B and ζ_Z the respective diffusion terms.

The use of discount factors is well-established in finance, and asset pricing literature. Smith and Wickens [2002] emphasize that specifying the discount factor appropriately can encompass many existing theories, including the Capital Asset Pricing Model (CAPM) and consumption-based CAPM frameworks. While single-factor models may be inadequate for capturing the term structure of prices, multi-factor and latent variable models offer a richer framework for interpreting pricing dynamics, as described by Cochrane [2005]. Renowned models such as those of Cox et al. [1985] and Vasicek [1977] have modelled discount factors as stochastic variables (mean-reverting diffusion processes) while this application of discount factors in real-options frameworks has been advanced by Thijssen [2011].

Let filtration $\{F_t\}_{t\geq 0}$ represent all the information generated by observing the path of the Brownian motions up to time t. With no arbitrage opportunities on the traded markets, we get :

$$E[d\lambda S_t|\mathcal{F}_t] = E[d\lambda R_t|\mathcal{F}_t] = 0, \forall t \geq 0$$

[Cochrane, 2005] then shows that this is the case if $\mu_\lambda = r$ and $\zeta_z = h_S$ with $h_S = \frac{\mu_S - r}{\sigma_S}$, the Sharpe ratio of asset S. From this result, it stems that:

$$\frac{d\lambda_t}{\lambda_t} = -rdt - h_S dB - \eta dZ$$

With $\eta \in \mathbb{R}$, and η interpretable as the price of the idiosyncratic risk. However, since the market is incomplete, there are an infinity of discount factors of the following form that price this risk. Hence, η could take any value in the case of an incomplete market.

Let us now introduce a probability measure $\mathcal{Q}$, observable on the measurable space $(\Omega, \mathcal{F})$. The NPV of a project priced with a stochastic discount factor is then:

$$\mathbb{E}^{\mathcal{Q}}(\pi^c) = \mathbb{E}^{\mathcal{Q}}[U - \int \lambda_\tau P_\tau d\tau]$$

$$= U - \mathbb{E}^{\mathcal{Q}}[\int \lambda_\tau P_\tau d\tau]$$

Given the undefined nature of our discount factor λ_t, one cannot simply solve a profit optimisation problem in this setup. We therefore introduce the notion of ambiguity to be able to solve this infinite-timed irreversible investment problem.

As highlighted by [Knight, 1921], ambiguity has to be distinguished from risk by taking into consideration that while risk is priced in a one-dimensional setting using a given probability measure $\mathbb{Q}$, ambiguity revolves around recognising that uncertainty leads investors to doubt on the choice of the probability measure that they should use. Here, we account for this ambiguity through the stochastic discount factor. While the manager is able to get a reference measure from past market observations, it has to consider perturbations around his reference measure. Since we are in an infinite-time framework, we use the particular case of κ-*ignorance* where these probability measures are included in a compact interval $[\kappa_{min}, \kappa_{max}]$[4]. κ_i hence determines the width of the interval, and is unique to each agent as information asymmetry comes into play.

We then get a set of measures $\mathcal{Q}^K$, with K a set of density generators (as defined in [Chen and Epstein, 2002]), that incorporates all probability measures in the aforementioned interval. While this is a more restrictive case (we only consider a smaller set of probability measures), this will allow us to derive analytical results, as the optimisation problem becomes linear. [5]

In this set of measures, [Thijssen, 2011] shows that there exists a measure $\mathcal{Q}_i^*$ for each agent i such that:

$$NPV_A^p = -I + \mathbb{E}_t^{\mathcal{Q}_p^*}\left(\int_t^{t+T} e^{-r(\tau-t)}P_\tau d\tau\right)$$

$$NPV_A^c = U - \mathbb{E}_t^{\mathcal{Q}_c^*}\left(\int_t^{t+T} e^{-r(\tau-t)}P_\tau d\tau\right)$$

Under this measure $\mathcal{Q}_i^*$, the price $(P_t)_{t\geq 0}$ has the stochastic differential equation:

$$\frac{dP}{P} = \mu_i^* + \sigma_B dB^* + \sigma_W dW^*$$

With $\sigma_B = \sigma_P \rho$, $\sigma_W = \sigma_P\sqrt{1-\rho^2}$, and

$$\mu_i^* = \mu - \sigma_P\left(\rho h_S + \sqrt{1-\rho^2}(\hat{\eta} + \kappa_i^*)\right) \tag{8}$$

B^* and Z^* are two independent Brownian motions defined for all $t \geq 0$.

This result entails that, for a given set of priors, the investment manager approximates the price of the idiosyncratic risk through the variable $\hat{\eta}$. However, the ambiguity caused by market incompleteness leads the manager to doubt of the appropriate probability measure. Ambiguity aversion then leads the decision-maker to consider investments only through the lens of the worst imaginable probability measure (see Gilboa and Schmeidler [1989] for formalisation), whose prior is contained in the interval $[\hat{\eta} - \kappa_{min}, \hat{\eta} + \kappa_{max}]$. These bounds can usually be interpreted as the minimal and

[4] One could extend these results in a finite framework, as Novikov's condition would then hold

[5] This is a great comparative advantage compared to the approach used in [Henderson, 2007], which finds comparable results by solving a non-linear Bernoulli equation problem

maximal Sharpe ratios that the agents consider possible on the non traded asset P, stemming from the non-hedgeable idiosyncratic risk Z_t, once the market risk has been priced. The intuition behind such bounds - referred to as "Good Deal bounds" in [Cochrane, 2005] - is that, if a higher (resp. lower) Sharpe ratio were to appear, it would constitute such a "good deal" that investors would be quickly attracted, rule out the arbitrage opportunity and re-equilibrate the market towards a lower (resp. higher) Sharpe ratio (see details in C).

Both agents are ambiguity-averse. They will therefore choose the prior $\kappa_i^* \in [\kappa_{min}^i, \kappa_{max}^i]$ that minimises their respective expected returns. The producer's (resp. the consumer's) return is decreasing (resp. increasing) with the prior κ. Thus, the profit's its profits will be minimised by taking $\kappa_p^* = \kappa_{max}^p$ (resp. $\kappa_c^* = \kappa_{min}^c$). By defining $\kappa_p := \kappa_{max}^p$ and $\kappa_c := -\kappa_{min}^c$, one can extend or reduce the prior segment to a symmetric one $[-\kappa_i, \kappa_i]$ without loss of generality, in order to keep the notation from in [Thijssen, 2011].

Replacing this value of κ_i^* in equation 8, we then have:

$$\mu_p^* = \mu - \sigma(\rho h_S + \sqrt{1-\rho^2}(\hat{\eta} + \kappa_{max}^p))$$

$$\mu_c^* = \mu - \sigma(\rho h_S + \sqrt{1-\rho^2}(\hat{\eta} + \kappa_{min}^c))$$

Note that when applying the model to real-world case studies, agents may operate in different markets and thus exhibit different values of h_S, as well as varying correlations with the market for the traded good. In our case study - featuring, respectively, an electricity utility firm and an industrial consumer active in electricity markets - we expect the correlation to be positive for the producer and negative for the consumer.

And finally, the risk-adjusted NPV of the projects in the *investing alone* case is the expectancy of the stochastic cash-flows under the measure of probability $\mathcal{Q}_i$ of each investor.

$$NPV_A^p(P_t) = -I + K(r - \mu_p^*)P_t \tag{9}$$

$$NPV_A^c(P_t) = U - K(r - \mu_c^*)P_t \tag{10}$$

2.4 Benchmark model : Completing the market with a Bilateral Contract

In this section, we derive a benchmark model were both agents take their decision following an *NPV criterion*. The purpose of this first benchmark model is twofold:

- First of all, this parsimonious model mimics the main mechanisms behind most static equilibrium models of incomplete markets. These models often do not consider the irreversible nature of investments: this benchmark will hence serve as reference to quantify the effects of irreversibility.

- Second of all, it might help the reader to build intuition on a simple case before the next section, where the same analysis will be carried on in a Real Options framework.

The risk-adjusted NPV of each agent i can be modelled as the expectancy under the natural probability measure of his stochastic NPV (as derived in previous sections) :

$$NPV_A^i = -I + K(r - \mu_i^*)P_t \qquad \text{for the investment alone} \tag{11}$$

$$NPV_{BC}^i = -I + X \qquad \text{for the investment with a Bilateral Contract} \tag{12}$$

These NPV values are the risk-adjusted, expected values investors might earn once they have invested in a real asset, with or without contracting. However, when these NPV are non positive (i.e. when X or P_t are too low for the producer), investors can postpone their investment. Taking this possibility into account and reasoning under the *NPV decision criterion*, one can then define the *investment opportunity NPV* $F_{NPV}^i(P_t, X)$ for each agent i, defined for all possible state of the market $\{P_t, X\}$:

$$F_{NPV}^i(P_t, X) = Max \begin{cases} NPV_A^i(P_t) = U - K(r - \mu_c^*)P_t & \text{investing alone} \\ NPV_{BC}^i(X) = U - X & \text{investing with a BC} \\ 0 & \text{delaying investment} \end{cases} \tag{13}$$

Note that, under the *NPV decision criterion*, delaying the investment is equivalent to doing nothing, hence delivering a value of 0. In other words, we do not consider the value of the option to wait yet.

For each couple of market and contract prices $\{P_t, X\}$, the maximal value defining F^i will be reached by one of the three expressions. Using this, we can define three regions of the plane $\{P_t, X_t\}$: the '*Invest Alone*', '*Bilateral Contract*' and '*Wait*' zones. In each region, F will be defined by the corresponding expression. From an economic point of view, drawing each region allows us to identify the best decision for the decision maker, depending on P_t and X_t : investing alone, investing with a PPA, or waiting.

For example, the producer will:

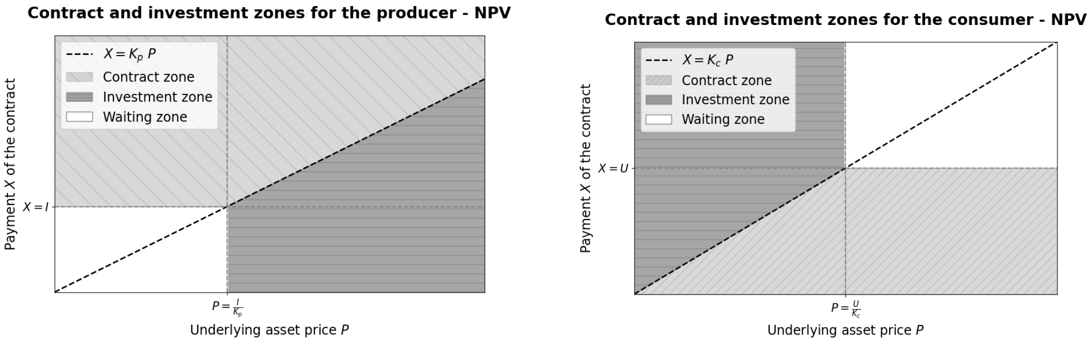
- *Invest Alone*: The producer will invest without hedging if the expected NPV from investing alone is positive and greater that the expected NPV with a Bilateral Contract. This corresponds to the zone where $\{X < K(\circ)P\}$.

- *Bilateral Contract*: In the remaining space, a bilateral contract will be signed if the BC NPV is positive and greater than the investment alone NPV. This corresponds to the zone $\{X > K(\circ)P\}$.

- *Wait*: In the region $\{P_t < I/K(\circ), X < I\}$, investing alone and investing with a PPA are both non-profitable. In this region, the investor will wait.

The decision rules of the consumer under the NPV criterion can be defined in the same way.

This simple analysis allows us to introduce the following representations of each regions, depending on the asset and contracts prices P_t and X :



(a) Producer

(b) Consumer

Figure 1: NPV investment regions: BC signature (light grey) and Investment alone (dark grey)

On these two plots, two important elements can be noticed :

- $U > I$: the joint investment is socially profitable
- the operator $K(\circ)$ is larger for the consumer than for the producer, leading to a steeper slope $X = K^i P$ for the consumer. This difference appears because each agent will choose a different worst-case probability measure to price the idiosyncratic risk : the producer (resp. the consumer) will expect a lower (resp. higher) increase in future asset prices than under the natural probabilities.

These two statements allow the apparition of a zone of possible agreement to sign a BC between both agents. This zone corresponds to the intersection of both agent's BC regions, as represented on Figure 2.

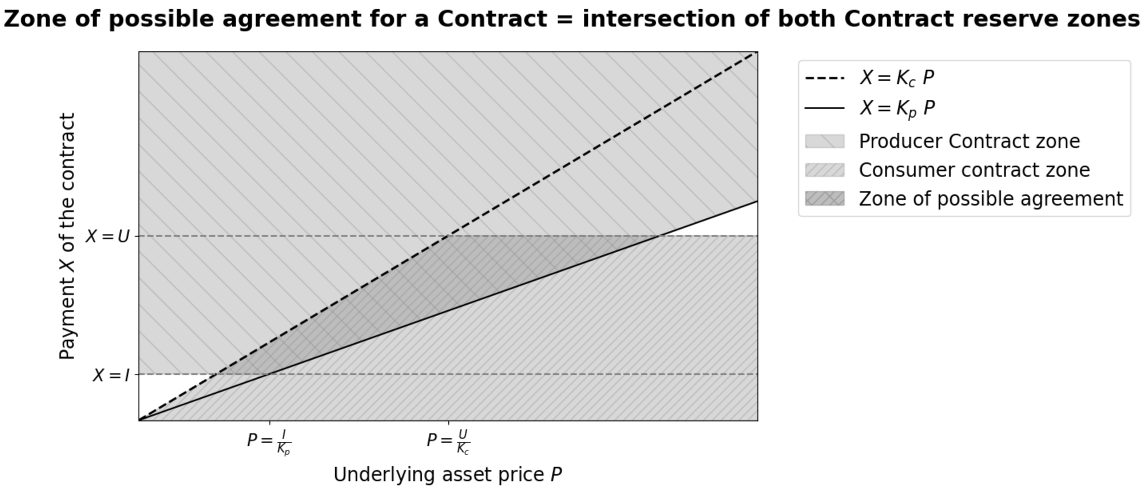


Figure 2: Overlay of Contracting Zones
The two regions where each agent is willing to enter a Bilateral Contract are shadowed. Their intersection, appearing in the deepest shade of grey, is the zone of possible agreement where a BC can be signed.

2.5 Accounting for Irreversibility

In this work, we relax the usual hypothesis around reversibility and focus on *irreversible* investments. In traditional project valuation, the decision to invest is often framed as a binary choice: either proceed with the project and reap the expected profits or abandon the investment, resulting in a payoff of zero. This framework, while useful, overlooks the value of managerial flexibility[6] in the face of uncertainty. Real option theory expands this framework by recognizing that investment opportunities often carry a third possibility: the option to wait. Rather than assigning a value of zero to delayed investments, real option valuation assigns a positive value to waiting, reflecting the potential to gain more information or capitalize on more favourable conditions in the future ([Dixit and Pindyck, 1994]).

If they invest, the investment value is of course the same as the one computed above : $\mathbb{E}^{\mathcal{Q}_i^*}(\pi^i)$. If investors invest as soon as the risk measure of their stochastic NPV is positive, they can expect to be profitable *in average*. But they can still endure a net loss ex-post - if the real asset price reveals to be lower than expected for the producer, for instance. By waiting before investing, agents postpone their potential benefits in time, but they gain information on the market :

- If the price has fallen in the meanwhile, the producer would have suffered a loss if he had

[6] Following the literature on real options valuation, we consider the value of managerial flexibility as the equivalent of the irreversibility premium

invested before.

- If the price has risen, the producer will then invest and gain the profit he would have had investing sooner, with a slight time-discount.

Should adverse events be possible and the discount rate sufficiently low, the value of waiting will be higher than zero. Using the real option framework introduced by [Dixit and Pindyck, 1994], one can compute the overall value of the *option to invest* F_i, which depends on the asset price P_t at the time of decision t (where $i = p$ or $i = c$ for the producer or consumer). In their landmark book, Dixit and Pindyck do not consider the contracting option. They then write:

$$F_i(P_t) = Max \begin{cases} \mathbb{E}_t^{\mathcal{Q}_i^*}(\pi^i(P_t)) & \text{investing} \\ \mathbb{E}_t^{\mathcal{Q}_i^*}\left(e^{-rdt}F_i(P_{t+dt})\right) & \text{waiting } dt \end{cases} \tag{14}$$

2.5.1 Valuing the Option to Invest

Similar to the previous benchmark model, each agent has three options at any given time t: *invest alone*, *invest with a BC*, or *wait* before making a decision. However, unlike the NPV criterion, where the value of waiting was zero, this is no longer the case in a dynamic framework where agents make optimal decisions under irreversible investment hypotheses. In this context, the value of waiting is defined as the discounted, risk-adjusted expectation of the future value of the investment opportunity:

$$\mathbb{E}_t^{\mathcal{P}}\left(\lambda_{t+dt}F^i(P_{t+dt}, X)\right) = \mathbb{E}_t^{\mathcal{Q}_i^*}\left(e^{-rdt}F^i(P_{t+dt}, X)\right) \tag{15}$$

Following the same steps as in the benchmark model, one can then define the option value F^i of the investment :

$$F^i(P_t, X) = Max \begin{cases} NPV_A^i(P_t) & \text{investing Alone} \\ NPV_{BC}^i(X) & \text{investing with a BC} \\ \mathbb{E}_t^{\mathcal{Q}_i^*}\left(e^{-rdt}F^i(P_{t+dt}, X)\right) & \text{delaying the investment} \end{cases} \tag{16}$$

As in the NPV case, one can define three regions $\{P_t, X\}$, where it is respectively optimal to invest alone, sign a BC, or wait. They respectively correspond to market state where F^i is equal to the first, second or third expression of 16.

Let us first provide some intuition regarding the shapes of these three regions, using the NPV benchmark model as a reference. In the benchmark model, an agent decides among three options: *investing alone*, *investing with a BC*, or *waiting*. However, in this *real option* framework, the option to wait is now recognized as a valuable alternative. Consequently, agents approach their decisions with greater caution, potentially opting to wait an additional period rather than rushing to conclusions.

When facing irreversible investment decisions, acknowledging the positive value of waiting impacts both the investment-alone and contracting boundaries. Both agents will rather wait than invest if the market state is too close to a frontier - allowing them to take into account the possibility of a change in market state at $t + dt$. For the investment-alone option, agents may choose to wait until market prices rise further, rather than investing immediately as soon as the NPV becomes positive. Similarly, for investment with a bilateral contract, agents are likely to delay in order to negotiate and ensure the contract is favourable, rather than signing hastily as soon as the NPV of the investment with a contract turns positive.

Hence, one can expect the waiting zone to expand its frontiers further in both the *Bilateral Contract* and *Invest Alone* zones (compared to the NPV zones). More interestingly, a new waiting zone will appear around the frontier PPA/investment, to better handle the risk of choosing one option over the other.

By following the steps of Dixit and Pindyck [1994], and defining value matching as well as smooth pasting conditions for the option value, one can derive the equation implicitly characterising $P^*_{A,i}(X)$ - the investment alone threshold as a function of the contract price X (see Appendix A for the detailed calculations):

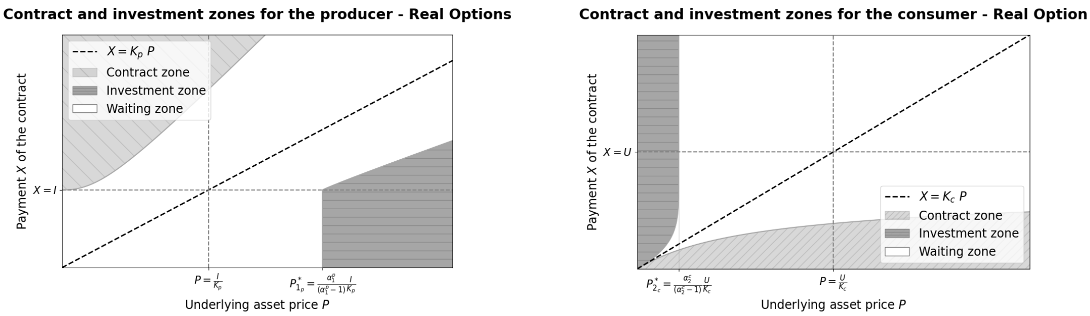
$$\begin{aligned} \frac{\left(\left(1-\frac{1}{\alpha_2^p}\right)K(r-\mu_p^*)P^*_{A,p}(X)-I\right)^{\frac{1}{\alpha_1^p}}}{\left(\left(1-\frac{1}{\alpha_1^p}\right)K(r-\mu_p^*)P^*_{A,p}(X)-I\right)^{\frac{1}{\alpha_2^p}}} &= (X-I)^{\frac{1}{\alpha_1^p}-\frac{1}{\alpha_2^p}} \\ \frac{\left(U-\left(1-\frac{1}{\alpha_2^c}\right)K(r-\mu_c^*)P^*_{A,c}(X)\right)^{\frac{1}{\alpha_1^c}}}{\left(U-\left(1-\frac{1}{\alpha_1^c}\right)K(r-\mu_c^*)P^*_{A,c}(X)\right)^{\frac{1}{\alpha_2^c}}} &= (U-X)^{\frac{1}{\alpha_1^c}-\frac{1}{\alpha_2^c}} \end{aligned} \tag{17}$$

This then allows to compute the contracting threshold frontiers with the expressions :

$$P^*_{BC,p}(X)^{\alpha_1^p} = \frac{X-I}{\left(\left(1-\frac{1}{\alpha_2^p}\right)K(r-\mu_p^*)P^*_{A,p}(X)-I\right)} P^*_{A,p}(X)^{\alpha_1^p} \tag{18}$$

$$P^*_{BC,c}(X)^{\alpha_1^c} = \frac{U-X}{\left(U-\left(1-\frac{1}{\alpha_2^c}\right)K(r-\mu_c^*)P^*_{A,c}(X)\right)} P^*_{A,c}(X)^{\alpha_1^c} \tag{19}$$

As usually highlighted in the Real Option theory, the irreversible nature of investments leads to an irreversibility premium being asked before any investment or contract signature. The waiting zone is larger than in the NPV case, as can be seen on Figure 3.



(a) Producer (b) Consumer

Figure 3: RO investment regions : Bilateral Contracts signature (light grey) and Investment alone (dark grey) zones

Finally, if one overlaps the reserve zones to sign a BC for both the producer and the consumer, the zone of possible agreement can, under some conditions of high volatility and relative low ambiguity, totally disappear, as in Figure 4.

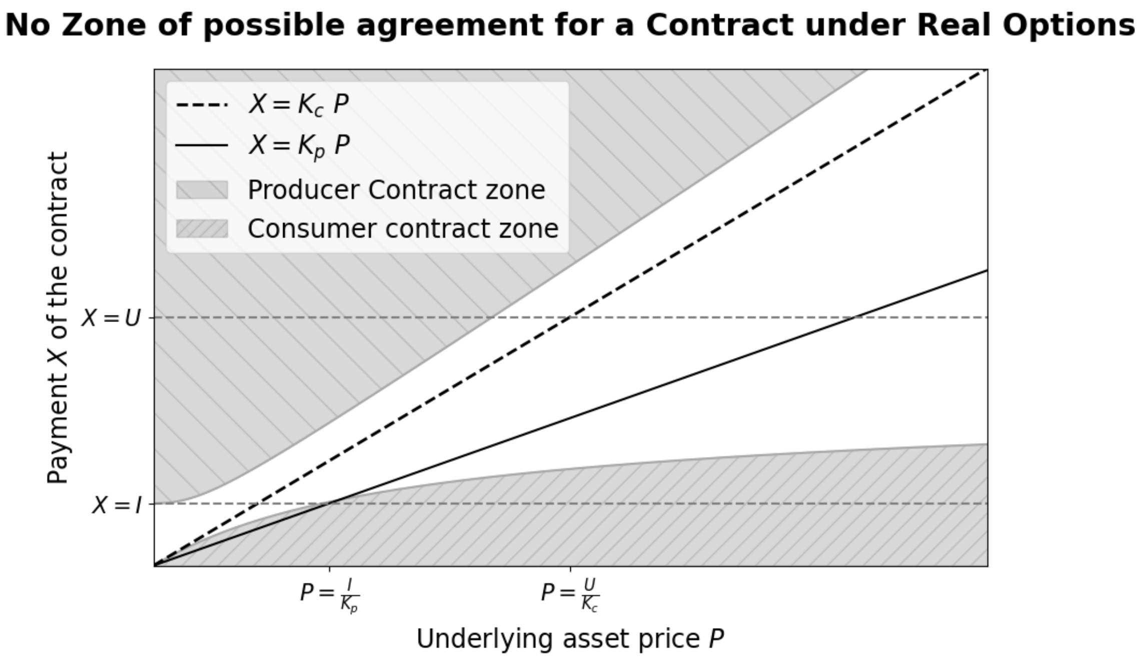


Figure 4: The two regions where each agent is willing to enter a BC are shadowed.
Contrary to the NPV benchmark case, the zone of possible agreement has totally disappeared.

An increase in volatility leads to two opposing effects: a greater incentive to hedge due to risk aversion, but also a stronger real option effect; depending on which effect dominates, this determines whether a contract is signed or not, highlighting the importance of a sensitivity analysis.

2.6 Sensitivity Analysis

The impact of the worst possible prior κ - which models the ambiguity aversion or idiosyncratic risk aversion of the agents - is primordial on the possibility to sign a contract. In the NPV model, we have seen that, as soon as the agents were slightly averse to ambiguity (or equivalently to the idiosyncratic risk), it was mutually beneficial to sign a BC, so that both agents were able to hedge against the market volatility. With irreversibility however, we show that it is not necessarily the case, and that the agreement zone can totally disappear due to the volatility effect, which creates opportunity costs from signing the contract. However, for higher values of κ, the expectation of future electricity prices becomes different enough between both agents to make a zone of possible agreement appear again. This effect is quantified in Figure 5[7] :

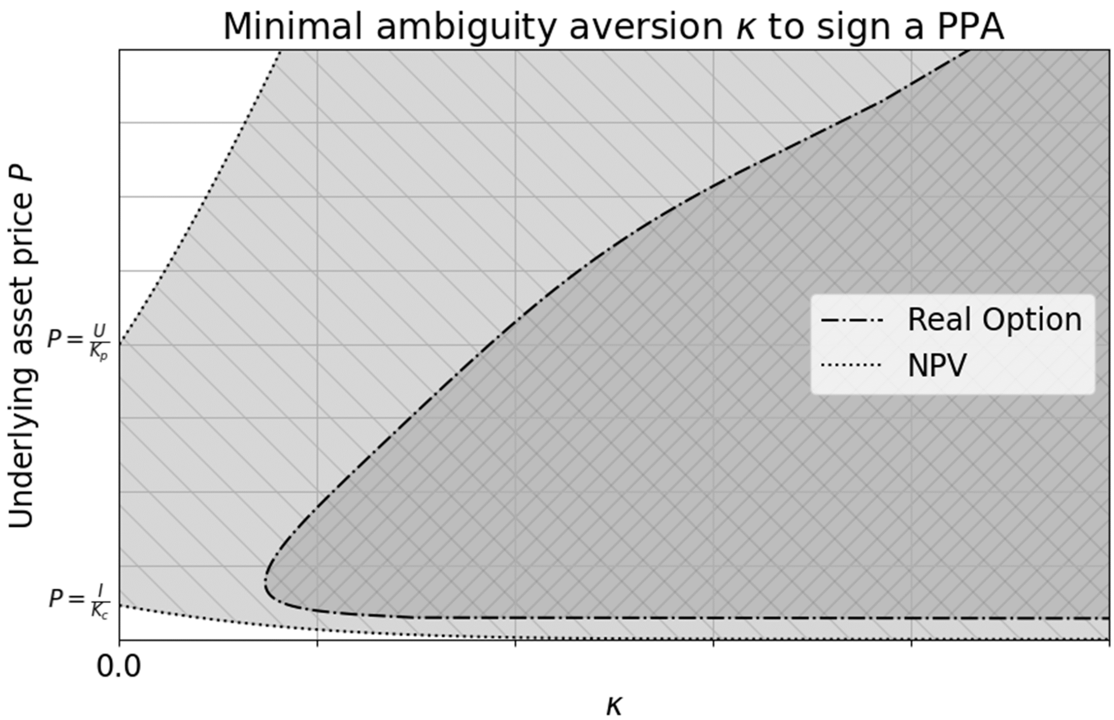


Figure 5: Under the usual NPV decision criterion, agents can sign PPA even if they are ambiguity-neutral regarding the idiosyncratic risk Z_t. Taking into account the irreversibility of investments however, PPA can be signed only if agents are ambiguity-averse enough.

[7] To simplify interpretation for our readers, we here consider symmetric ambiguity parameters $\kappa_m in$ and κ_{max}. In a real world scenario, these parameters might not be symmetric.

We were able to demonstrate that, contrary to the intuition based on a static NPV model, new entrants cannot necessarily sign a BC to isolate from the non-hedgeable risk on the electricity market and invest as if the market were complete. However, this result depends on the relative strengths :

- of the ambiguity aversion on one side, pushing agents to sign a BC when it is large enough ;
- And of the market volatility and characteristics on the other side, preventing agents to sign contracts when they consider irreversible investments.

Now referring to Figure 6, we observe the minimum ambiguity levels required to sign a bilateral contract, depending on the real asset price P and market volatility. For reasonable values of P, an increase in volatility implies that a higher minimum ambiguity level is needed to allow for a BC. This finding reinforces our earlier observations: greater volatility extends the waiting zone, prompting investors to adopt a more cautious approach, as the value of future information increases with σ_P. The competing effects of the real options and ambiguity criteria, discussed in the previous section, are put forward. As volatility rises, the value of waiting increases, thus requiring a higher ambiguity level to reach an agreement.

However, this logic reverses at extreme values on either end of the figure. When the price is very high and the volatility low, producers are inclined to invest alone, as their expected NPV far exceeds the acceptable contract prices offered by consumers. Symmetrically, in cases where both market prices and volatility are low, the consumer similarly prefers to invest independently. Only in scenarios of extremely high ambiguity can this effect be counteracted - as severe ambiguity leads producers to heavily discount future risky cash flows. Under such conditions, they may prefer the stability of steady income over almost-certainly higher, but risky, returns.

When the volatility goes to zero, the behaviour of this minimal ambiguity aversion to sign depends on the drift of the electricity price, and is studied extensively in Appendix D. Here, it can be shown that for very low volatilities, agents will sign whatever their ambiguity aversion for prices between two boundaries I/K and $\frac{r}{r-\mu}\frac{U}{K}$, and never sign for other prices.

To conclude on the possibility of signing a BC and the significance of each effect, one has to calibrate the model on a real case study. This is precisely the objective of the next section.

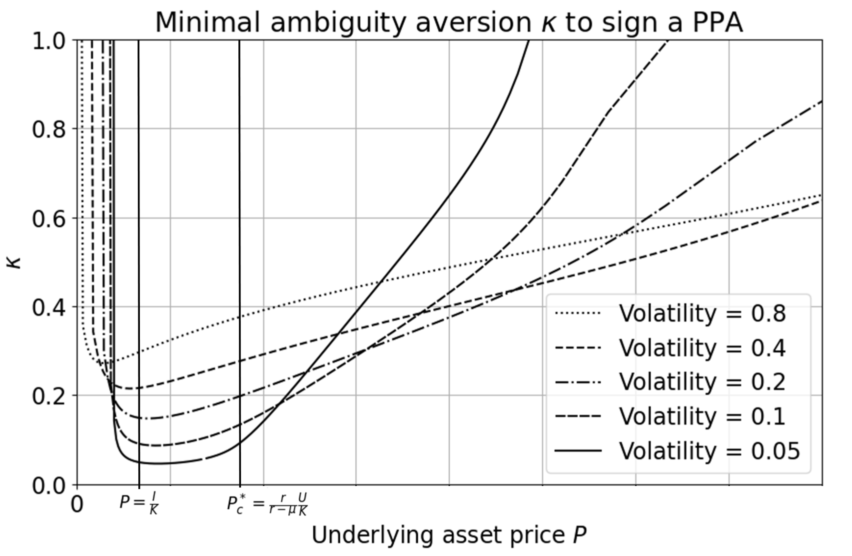


Figure 6: Minimum Ambiguity Levels to sign a contract

3 Empirical Case Study

In this section, we calibrate our model using EU data to explore a real-world application and provide concrete insights into our research question. We choose study ongoing negotiation between EDF, the historical French electricity producer, and ArcelorMittal, a major industrial company. This real-world case provides an ideal setting to assess the practical application and effectiveness of our theoretical model. As these negotiations are public, they offer a transparent view into the dynamics at play, enabling us to test the validity of our approach. We will also complement this main case study with a second one, looking at potential PPA signings between ArcelorMittal and EDF's historical capacity can be signed, given the absence of investment costs for the French power producer. This second analysis will shed light on a complementary problem: if industrials are not able to sign PPAs with new power development projects, do they have the opportunity to do so with existing low-carbon capacities?

The focus of the negotiations aligns closely with the central themes of our model: the need for the electricity producer to secure stable revenues by locking in a fixed price for its electricity, and the industrial consumer's aim to stabilize operating costs. Specifically, ArcelorMittal's strategic shift from the traditional, high-emission Blast Furnace-Basic Oxygen Furnace (BF-BOF) process to the more sustainable Electric Arc Furnace (DRP-EAF) process underscores the critical role of low-carbon electricity in reducing emissions[8].

[8]Note that, in practice, firms with limited capital may have other available options, such as investing in another

Moreover, the ambiguity framework allows us to capture the lack of foresight regarding the severity, timing, and effects of future events. This framework has been increasingly adopted in the literature on climate change policymaking due to its relevance to uncertain environmental damages and regulatory shifts [Millner et al., 2013, Heal and Millner, 2013, Berger and Marinacci, 2020].

3.1 Assumptions

To model electricity prices over the long term, we adopt a Geometric Brownian Motion (GBM) framework. More precisely, and to avoid the explicit modelling of short term jumps and mean reversion, we calibrate the price P as the sum of electricity prices over one year (or $\frac{P}{8760}$ as the yearly average electricity price). It is this average price that follows a GBM in our calibration. While short-term electricity prices are typically modelled as mean-reverting processes, such as Ornstein-Uhlenbeck dynamics, long-term have been modelled with GBM in multiple instances([Schwartz and Smith, 2000, Cheng et al., 2017, Takizawa and Suzuki, 2004, Cortazar et al., 1998, Compernolle et al., 2022]). Moreover, Pindyck [1999] that the low rate of reversion in energy prices over the long-term makes using a GBM for electricity price modelling a reasonable approximation for investment analysis. This supports our choice to retain GBM in our model.

In our empirical evaluation, we deliberately focus on the decision-making process surrounding the Power Purchase Agreement (PPA) signing[9]. PPAs are bilateral contracts that allow electricity producers and consumers to agree on a fixed price for a specific quantity of electricity over a defined period. They provide financial stability in volatile markets, lowering the Weighted Average Cost of Capital (WACC) for producers and facilitating access to low-cost financing [May and Neuhoff, 2021, Gohdes et al., 2022, Kapral et al., 2024]. As renewable energy penetration increases, particularly from intermittent sources like wind and solar, PPAs become crucial for ensuring stable cash flows and reducing exposure to price volatility.

We assume a minimum demand level that ensures full consumption of the electricity acquired through these agreements. Additionally, we have chosen this case to eliminate counterparty risk from our analysis. In negotiations involving smaller companies, such risks might need to be internalized through a stochastic model, potentially introducing new avenues for research in real-options-based contracting models. However, in this instance, the robust financial standing of both parties removes this variable, allowing us to focus purely on the dynamics of the PPA negotiation. Finally, we make the assumption that ArcelorMittal is here covered by a CCfD scheme allowing it to hedge

project or delocalising. In this case, as the negotiations have been launched and subsidies granted, we put these alternatives aside.

[9] All other things equal. We assume that carbon risk is stable or hedged; scrap price fluctuations are not to take into account as they would have the same impact on carbon-intensive and low-carbon production

its exposure to carbon price risk when decarbonising, allowing us to concentrate on electricity market implications (and allowing us to also comment on the anticipated efficiency of such instruments).

3.2 Calibration

3.2.1 Case Specific

In order to correctly calibrate our model, we first need to compute the project-specific costs of investment I for the producer as welle as the consumer's net value of the investment U. Using company disclosures and announcements, European regulations press releases, as well as online French articles from reliable media sources, we are able to reconstruct the investment costs estimations and expected value of investment for both parties. As the two projects have different lifetimes and power capacities, we normalise their Net Present Costs [10] per MWh by first dividing them by the discounted energy consumed/produced. This allows us to compare the two values on an appropriate common scale. The investment cost in the generation unit takes the form of the well-known Levelized Cost Of Electricity (LCOE), while the net investment value in the steel mill can be considered as an analogous Levelized Revenue Of Electricity Consumed (LROEC) :

$$LCOE = \frac{I_p + \sum_y (1+r_p)^{-y} VC_y}{\sum_y (1+r_p)^{-y} MWh_{p,y}}$$

$$LROEC = \frac{-I_c + S + \sum_y (1+r_c)^{-y} \left(Ab_y . P_{CO2,y} + C_y\right)}{\sum_y (1+r_c)^{-y} MWh_{c,y}}$$

We then compute the annualized sum of these values on the 15 years lifetime of the contract to find the equivalent upfront utility and cost U and I of our model :

$$U = LROEC \times \sum_{y=1}^{15} (1+r_c)^{-y}$$

$$I = LCOE \times \sum_{y=1}^{15} (1+r_p)^{-y}$$

With $I_i > 0$ the investment costs[11] for agent i, $MWh_{i,y}$ the expected production (respectively consumption) in megawatt-hours during year y. All cashflows are discounted with agent i 's exogenous cost of capital r_i. We assume this cost of capital only prices market risks (stemming from the correlation of the stock with the market, and the stock's volatility), and not the idiosyncratic risk. The generation unit has yearly variable costs VC_y. Specific to the consumer, we have the yearly level of emissions abatement Ab_y, the EU ETS anticipated price in year y $P_{CO2,y}$, S the

[10] See Annex D for more details

[11] Integrating provisions for anticipated costs

	Value	Source
U	850 €/MWh.y	Appendix B
I	821 €/MWh.y	EDF

Table 1: Model Inputs

amount of government subsidies and C_y the other avoided costs (i.e. operating costs such as fossil fuel consumption).

By comparing the $LCOE$ and $LROEC$ to the yearly average electricity price, we implicitly abstract from the issue of coordinating electricity production and industrial demand. Our choice to focus on ArcelorMittal and EDF in the case study is partly motivated by their operational flexibility and working hours, which make this simplification more reasonable. Future research on mechanisms such as renewable generation pooling to better fit to industrial demand could help address this coordination challenge and extend our framework to PPAs between industrial consumers and renewable energy producers.

In this case study, subsidies are justified through positive externalities stemming from investment in low-carbon processes, from a social welfare point of view (avoided GHG emissions, learning spillover effect, sovereignty...). Table 1 gives the final results for both inputs.

3.2.2 Market Data

For our market data analysis, we focus on the European market electricity. This aligns with our decision to model the negotiations between EDF and ArcelorMittal. We retrieved non-household consumer average electricity prices from Eurostat [12]. Looking at market data, we downloaded Eurostoxx data from the past ten years on a monthly basis using the Refinitiv database[13] to calculate yearly market volatility and expected rates of return for ArcelorMittal, while we chose the MSCI Energy Utilities ETF to calibrate EDF's market data. Additionally, we used Refinitiv data to obtain the risk-free rate, opting for the European Euribor index, which is widely referenced in financial literature. All calibrations are available in B.

To compute mean rates of return and volatilities for the electricity, we compute the yearly log returns, and apply the Pearson volatility formula :

$$r_t = ln(x_n) - ln(x_{n-1})$$

$$\mu = \frac{\sum_{n=0}^{N} r_n}{N}$$

[12] available at https://ec.europa.eu/eurostat/databrowser/view/nrg˙pc˙205˙˙custom˙17222791/default/table?lang=fr
[13] Data can be accessed only through a paid Refinitiv subscription

σ_P	$\hat{\eta}$	κ_{min}	κ_{max}	μ_{min}	μ_{max}
0.22	0.091	-0.30	0.90	-0.026	+ 0.238

Table 2: Main Parameters

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{n=0}^{N}(r_n - \mu)^2}$$

with x_t the price level at time t, r_t^i the yearly rate of return corrected from the EURIBOR rate (or market to risk free spread), μ the average rate of return. All market data can be found in B.

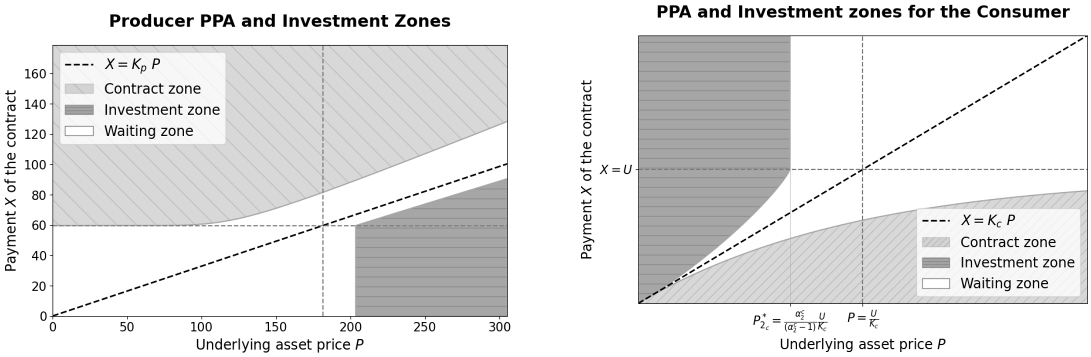
To identify κ_{min} and κ_{max}, we divide the full sample period into rolling windows of four years, corresponding to the maximum maturity of the most liquid electricity futures contracts, and compute the Sharpe ratio of the traded good in each window. We drop the assumption of symmetric ambiguity and instead adopt a more flexible calibration method, allowing for a more realistic characterization of ambiguity levels and aversions. We follow Thijssen [2011] in fixing $\hat{\eta}$ at the level of the Sharpe ratio of the real asset. The set of priors is then defined by deviations from this reference value, yielding the ensemble $\kappa = [\kappa_{\text{min}}, \kappa_{\text{max}}]$, which we calibrate using historical data. This allows us to capture the most extreme outcomes and define the bounds of the ambiguity set in the spirit of Gilboa and Schmeidler [1989]. The minimum and maximum Sharpe ratios observed across rolling windows are denoted by $\hat{\eta} + \kappa_{\text{min}}$ and $\hat{\eta} + \kappa_{\text{max}}$, respectively. We then derive κ_{min} and κ_{max} by subtracting $\hat{\eta}$ from these observed extremes, which represent the lower and upper deviations from the average Sharpe ratio over the full sample.

For ease of interpretation, we present the implied values of μ_{min} and μ_{max}, which can be viewed as the worst and best expected returns under ambiguity. Since κ enters linearly in the drift term μ^* (see Equation 8) as $\mu = r + \sigma_P(\hat{\eta} + \kappa)$, these values provide a direct interpretation of the cost of ambiguity in return terms.[14] The final calibrated values are reported in Table 2, and additional calibration details can be found in Appendix C.

3.2.3 Results

Referring to 7, we observe that incorporating electricity market parameters, particularly high volatility, significantly shrinks both the PPA and investment regions. Under conditions of high volatility, the irreversibility effect becomes dominant, leading to an expansion of the waiting zone.

[14] This calibration method allows us to directly link market data to ambiguity bounds, and to interpret $\mu_{\text{min}} = r + \sigma_P(\hat{\eta} + \kappa_{\text{min}})$ and $\mu_{\text{max}} = r + \sigma_P(\hat{\eta} + \kappa_{\text{max}})$ as the relevant limits of expected returns under model uncertainty.



(a) Producer

(b) Consumer

Figure 7: RO investment regions : PPA signature (red) and Investment alone (blue)

Now looking at 8, we see that the empirical analysis of the potential Power Purchase Agreement (PPA) between EDF and ArcelorMittal yields results that align with our initial hypotheses and forecasted outcomes. The findings highlight the intricate interplay between ambiguity aversion, the value of flexibility, and the economic feasibility of the investment for both parties involved. Given the very high volatility of European electricity markets, aggravated by the gas crisis, the opportunity costs of signing such a contract becomes too high for each of the agent, yielding a situation where counterparties are unable to find an agreement.

Looking back to our first scenario, where both agents were assumed to reason in an NPV framework, the analysis revealed that counterparties were be willing to sign a PPA as long as there was any ambiguity. The range of the overlapping PPA zones reflected the balance where each party felt sufficiently compensated for the risks and returns associated with the investment.

In our **main scenario**, calibrated on data ranging from 2009 to 2024[15], both irreversibility and ambiguity aversion are considered simultaneously. While the latter reduces the threshold for the consumer (here, EDF) to sign a PPA, the former exerts a countervailing force. The high volatility in electricity markets, combined with the value of delaying investments, results in no viable zone of agreement between EDF and ArcelorMittal. The dominance of the irreversibility effect, due to high market volatility and opportunity costs of committing, ultimately leads to the conclusion that a PPA is unlikely under these conditions, as neither party finds the terms favourable enough to commit to the investment. This holds because the level of price volatility raises the value of flexibility, and pushes both agents to postpone investment.

In order to hedge from electricity price risk and follow-up on its decarbonisation project, we estimate that ArcelorMittal would need to find a contractor that would be ready to sign a PPA

[15] Note that this rather restricted time-frame is justified by the short existence of liberalised electricity markets

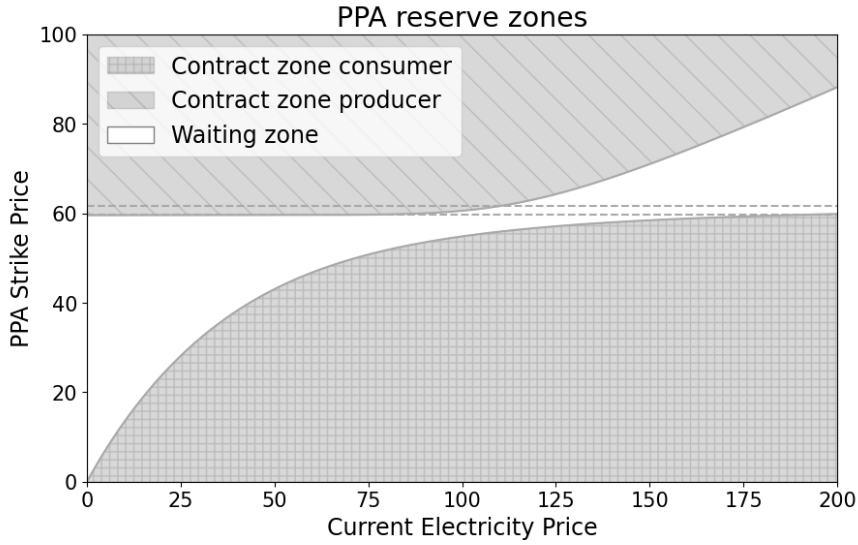


Figure 8: Empirical Effects of Risk Aversion and Flexibility Value

at around 50€/MWh [16] (assuming average yearly prices of around 60€). This results seems in line with current negotiations and previous long term contracts provide by historical French nuclear power plants. However, a rational EDF leadership should refuse to sign a PPA contract at this price range, except if market prices are already extremely low (making the probability of higher prices low).

Overall, these scenarios illustrate the complex decision-making processes underlying large-scale investments in decarbonised infrastructure. The interplay between uncertainty, flexibility, irreversibility and market dynamics plays a crucial role in shaping the outcomes of such negotiations, with significant implications for the viability of future PPAs in similar contexts.

We also compute the equilibrium contracting zones from 2009 to 2020, hence not accounting for the Russian gas crisis [17]. By only accounting for this time period, we make sure not to incorporate the Russian gas crisis in our data. We calibrate all parameters using the same methodologies. We then obtain the following parameters from 3. In this pre-crisis period, the cost of ambiguity for both agents μ_{min} and μ_{max} are smaller than in the crisis period. This diminishes the incentive to sign a PPA for both agents. But the lower volatility also reduces the opportunity cost of signing an irreversible contract. As shown in section 2.6, this second effect is preponderant, allowing for an easier signature of the contract, but for a narrower range of prices on the market.

[16] To convert the total PPA cash flows X into an interpretable strike price,

[17] Appendix 2.6 provides another sensitivity analysis for this case study, studying the minimal ambiguity aversion level required to sign a contract, depending on different volatilities and market prices.

σ_P	$\hat{\eta}$	κ_{min}	κ_{max}	μ_{min}	μ_{max}
0.055	0.078	-0.82	0.75	- 0.02	+ 0.065

Table 3: Markets Statistics

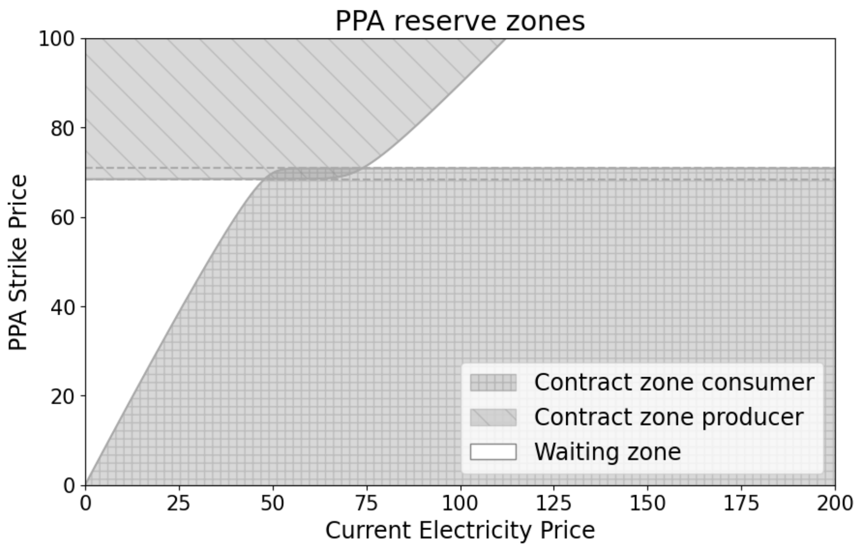


Figure 9: Pre-Crisis Calibration

Figure 9 shows that, if ArcelorMittal and EDF had begun their negotiations before the energy crisis, they could have reached an agreement on the strike price of their contract. This would have led to lower cost of capital for both companies, unlocking investments in new electricity generation and industrial decarbonization. Long-term contracts can hence improve welfare during periods of crisis by providing price stability, but we expect firms to struggle to reach agreements in times of heightened market fluctuations, as the opportunity costs of signing a long-term contract become too high to commit (most notably for the electricity producer in our case study). Long-term contracts should therefore be signed in times of price stability, as insurances against potential future crisis, and not seen as a last resort instruments once the crisis hits.

4 Discussion

This analysis provides a novel perspective on contract negotiations by integrating both flexibility and aversion to uncertainty into the decision-making framework of two investors with symmetric profit functions. By doing so, we offer a new approach to analyse contractual agreements within a general framework. Traditional models often overlook the interplay between these factors, but our analysis demonstrates how critical they are in shaping investment decisions, particularly in volatile incomplete markets such as the gross electricity market. This approach could serve as a foundation for further research in contract theory, particularly in contexts where uncertainty and strategic timing play significant roles.

This leads to the conclusion that isolating from an incomplete market is not an easy task. While one could think that incomplete markets could be completed by simply offering a platform for exchange, we find that the unpriced idiosyncratic risk entails difficulties in finding long term contractual agreements. By accounting for the irreversibility of investments, we open the discussion towards new policy incentives that would be aimed at fostering long term contracting between agents with symmetric profit functions. While one could think that the market's high volatility would only push agents to hedge, we show that this might not be the case if we account for the irreversible nature of decarbonisation investments.

Despite early expectations from governments and analysts that the PPA market would grow rapidly, the actual number of signed contracts has remained disappointingly low. Our findings suggest that this can be attributed to the combined effects of high market volatility and the strategic value of flexibility, which together discourage spontaneous investment. This insight is crucial for understanding the discrepancy between anticipated and actual market outcomes. Moreover, the issue of electricity market incompleteness emerges once again as a significant barrier to investment. The inability of agents to hedge against long-term electricity price risks creates a challenging environment where investments are rarely made without substantial government subsidies or the security of long-term PPA contracts. This market imperfection limits the potential for private investments in decarbonised infrastructure, highlighting the need for policy interventions that could address these gaps, such as enhancing market mechanisms for long-term risk hedging or providing additional financial support for critical projects.

Given our results, and considering that only renewable energies achieve sufficiently low LCOEs, they may be the only suitable counterpart to decarbonising industrials. A contract aggregating different renewable energy sources, maintaining a sufficient power generation capacity at all time, or the signature of financial PPAs with renewable energy providers, might be the way ahead. The modelling of these aggregated players, their LCOEs, and their willingness to sign long-term contracts makes for an interesting avenue for future research. Alternatively, by promoting risk-sharing through joint ventures and vertical integration between offer and demand, one could ensure the transition and security of supply of these strategic sectors while limiting their exposure to significant fluctuations driven by extreme events. Given the current geopolitical and climate uncertainties, such events are likely to become more frequent, making it all the more crucial to minimise risk in the energy transition.

Our findings suggest several key policy recommendations. Firstly, we remind that market incompleteness is symptomatic of deeper market failures. These failures cannot always be resolved simply by introducing new financial derivatives; instead, it is crucial to understand and address the frictions causing this incompleteness. A case in point is the failure of new energy derivatives to gain traction on platforms like EPEX Spot. While many theoretical arguments for market incompleteness apply to various sectors, they do not fully explain why the electricity markets remain incomplete. We propose a new theoretical explanation : the irreversible nature of long-term investments in these markets creates a unique barrier to market completion.

Secondly, they confirm that traditional Carbon Contracts for Difference, which only hedge against carbon price risks, might be an insufficient tool when looking to incentivise industrials to pursue a decarbonisation pathway. This has significant implications for any country considering a CCfD programme alongside the implementation of an Emissions Trading Scheme. For example, Germany has taken a comprehensive approach by offering its industries a combination of CCfDs and electricity CfDs ([for Economic Affairs and Action, 2023]), which hedge against both energy and carbon price risks. In contrast, France might opt for a more limited, one-dimension CCfD scheme. This divergence may explain the ongoing state-led negotiations between EDF and ArcelorMittal. While these talks are expected to succeed, the French CCfD design could fall short of its objectives by overlooking the need to address electricity price risk.

This leads to a critical insight: simply introducing new contracts, like the French government attempted to, may have limited impact at best. Even if the contracts were to be introduced by a public authority, a market maker enabling these exchanges would have to be appointed, as discussed in [Schittekatte and Batlle, 2023, Billimoria et al., 2024]. With this model, we give additional reasons to consider this direction. Simshauser et al. [2015] also discussed the advantages of physical hedges through integrated generators-retailers in energy-only markets. We believe that generator-manufacturer integration could have similar effects, while also profiting from the co-benefits of their decarbonisation.

In the latest report on EU competitiveness [Draghi, 2024], authors report that despite the expected benefits, PPA contracts have struggled to gain traction in the EU due to several financial and market barriers. These include a lack of financial guarantees for counterparty risk, limited market risk appetite, companies' creditworthiness, and the complexity and lack of standardization in contracts. While hybrid and multi-buyer PPAs have been contracted in small volumes, their adoption remains limited - particularly in energy-intensive industries, where uptake is still in its early stages. He promotes potential market reforms that would lead to increased long term contracting, either between consumers (PPAs) or through state interventions (CfDs). When looking at EU legal restrictions on the signing of PPA contracts, this is already a step ahead, as PPAs have historically been pushed back by market supervisors for competition purposes. However, our model highlights the limitation of only focusing on the administrative costs of finding a counterpart - as we do not consider transaction costs. We conclude that direct state interventions, or deeper market reforms, might be the only tools available to stabilise electricity prices for decarbonisation.

5 Conclusion

In this paper, we introduce a novel framework for analysing contracting under uncertainty, which accounts for the ambiguity aversion of private agents as well as the option value of flexibility when faced with irreversible investment decisions. By generalizing the analysis of bilateral contracting, we aim to equip economic researchers with a versatile tool to explore contracting dynamics and potential inefficiencies.

Our bilateral contracting model contributes to several strands of research. First, it offers a new perspective on market incompleteness for infrastructure-heavy sectors by identifying new drivers explaining potential market gap. We then extend the theoretical insights with real-world calibrations, which allows us to determine whether the risk aversion or irreversibility effect dominate in electricity markets. Our findings suggest that high market volatility amplifies the irreversibility premium and the associated opportunity costs of entering contracts, thereby outweighing the effects of ambiguity aversion. By addressing these issues, we bridge a literature gap by linking research on industrial decarbonisation, electricity market incompleteness, and risk management.

While this paper provides a solid foundation for a new analytical approach, there is still potential for further research to enrich its realism and applicability. For instance, future studies could examine how consumer flexibility - such as production schedule adjustments - affects energy cost volatility. Incorporating asymmetric parameters for producers and consumers could yield more accurate representations of their strategic interactions. Lastly, relaxing the assumption that producers and consumers trade equal quantities of electricity could better capture real-world contracting complexities.

With these extensions, the framework presented here has the potential to further contribute to the understanding and resolution of contracting challenges in decarbonising industries, as well as other markets facing incompleteness challenges.

Appendices

A Derivation of the Real Option Thresholds

Following standard steps ([Dixit and Pindyck, 1994]), this leads to the Bellman equation :

$$\frac{1}{2}\sigma_P^2 \frac{d^2F^i}{dP^2}P^2 + \mu_i^* \frac{dF^i}{dP}P - rF^i = 0 \tag{20}$$

As this equation has no derivative with respect to the contract price X, X can be considered as a parameter, and the equation can be solved as an ordinary differential equation in P. This gives the general solution (with integrating constants $A(X)$ and $B(X)$ depending on X) :

$$F^i(P,X) = A(X)P^{\alpha_1^i} + B(X)P^{\alpha_2^i} \tag{21}$$

To determine the integration constants, one can distinguish two cases :

1. **If $X < I$ (resp. $X > U$ for the consumer)**, it will never be interesting to sign a PPA. In this case, the option value and the frontier to invest alone does not depend on X anymore : the frontier becomes a vertical line $P = P^*_{A,i}$, where $P^*_{A,p} = \frac{\alpha_1^p}{\alpha_1^p - 1}\frac{I}{K_p}$ (resp. $P^*_{A,c} = \frac{\alpha_2^c}{\alpha_2^c - 1}\frac{U}{K_c}$ for the consumer) is the classical real option threshold from Dixit and Pindyck [1994]. The

option value in the waiting region for the producer and the consumer are then :

$$F^p(P < P^*_{A,p}, X < I) = (K_p P^*_{A,p} - I)\left(\frac{P}{P^*_{A,p}}\right)^{\alpha_1^p} \quad (22)$$

$$F^c(P > P^*_{A,c}, X > U) = (U - K_c P^*_{A,c})\left(\frac{P}{P^*_{A,c}}\right)^{\alpha_2^c} \quad (23)$$

2. **If $X > I$ (resp. $X < U$ for the consumer)**, then the three regions (PPA, waiting and investing alone) will be separated by two frontiers $P^*_{BC,p}(X)$ and $P^*_{A,p}(X)$ (resp. $P^*_{BC,c}(X)$ and $P^*_{A,c}(X)$ for the consumer).

Each of these two frontiers is implicitly characterised by two conditions :

- A *value matching condition*, stating that the waiting value should equal the investment NPV on the frontier (with or without BC, depending on the region).
- A *smooth pasting condition*, enforcing the derivatives of the waiting value with respect to P to equal the one of the investment NPV on the frontier.

The lead to the following 4 boundaries conditions, which allow to determine the integration constants $A(X)$, $B(X)$ and the frontiers expressions $P^*_{BC,i}(X)$, $P^*_{A,i}(X)$:

1. Value matching on the PPA frontier :

$$A(X)\left(P^*_{BC,i}(X)\right)^{\alpha_1^i} + B(X)\left(P^*_{BC,i}(X)\right)^{\alpha_2^i} = NPV^i_{BC}(P^*_{BC,i}(X), X) \quad (24)$$

2. Smooth pasting on the PPA frontier :

$$\alpha_1^i A(X)\left(P^*_{BC,i}(X)\right)^{\alpha_1^i - 1} + \alpha_2^i B(X)\left(P^*_{BC,i}(X)\right)^{\alpha_2^i - 1} = \frac{\partial NPV^i_{BC}}{\partial P}(P^*_{BC,i}(X), X) \quad (25)$$

3. Value matching on the investment alone frontier :

$$A(X)\left(P^*_{A,i}(X)\right)^{\alpha_1^i} + B(X)\left(P^*_{A,i}(X)\right)^{\alpha_2^i} = NPV^i_A(P^*_{A,i}(X)) \quad (26)$$

4. Smooth pasting on the investment alone frontier :

$$\alpha_1^i A(X)\left(P^*_{A,i}(X)\right)^{\alpha_1^i - 1} + \alpha_2^i B(X)\left(P^*_{A,i}(X)\right)^{\alpha_2^i - 1} = \frac{\partial NPV^i_A}{\partial P}(P^*_{A,i}(X), X) \quad (27)$$

Replacing the NPV by their expressions for the producer p for instance, one has :

$$A(X)\left(P^*_{BC,p}(X)\right)^{\alpha_1^p} + B(X)\left(P^*_{BC,p}(X)\right)^{\alpha_2^p} = X - I \quad (28)$$

$$\alpha_1^p A(X)\left(P^*_{BC,p}(X)\right)^{\alpha_1^p - 1} + \alpha_2^p B(X)\left(P^*_{BC,p}(X)\right)^{\alpha_2^p - 1} = 0 \quad (29)$$

$$A(X)\left(P^*_{A,p}(X)\right)^{\alpha_1^p} + B(X)\left(P^*_{A,p}(X)\right)^{\alpha_2^p} = K(r - \mu_p^*)P^*_{A,p}(X) - I \quad (30)$$

$$\alpha_1^p A(X)\left(P^*_{A,p}(X)\right)^{\alpha_1^p - 1} + \alpha_2^p B(X)\left(P^*_{A,p}(X)\right)^{\alpha_2^p - 1} = K(r - \mu_p^*) \quad (31)$$

If one sums each value matching condition with its associated smooth pasting condition multiplied by $-\frac{P^*_{BC,A,p}(X)}{\alpha_2^p}$, the integration constant $B(X)$ cancels out and we get the following:

$$A(X) = \frac{X - I}{\left(1 - \frac{\alpha_1^p}{\alpha_2^p}\right) P^*_{BC,p}(X)^{\alpha_1^p}} \tag{32}$$

$$A(X) = \frac{\left(\left(1 - \frac{1}{\alpha_2^p}\right) K(r - \mu_p^*) P^*_{A,p}(X) - I\right)}{\left(1 - \frac{\alpha_1^p}{\alpha_2^p}\right) P^*_{A,p}(X)^{\alpha_1^p}} \tag{33}$$

And finally, by equalising both equations, one finds equation 18:

$$P^*_{BC,p}(X)^{\alpha_1^p} = \frac{X - I}{\left(\left(1 - \frac{1}{\alpha_2^p}\right) K(r - \mu_p^*) P^*_{A,p}(X) - I\right)} P^*_{A,p}(X)^{\alpha_1^p} \tag{34}$$

Following the same steps but keeping the constant $B(X)$ and cancelling $A(X)$, we get:

$$P^*_{BC,p}(X)^{\alpha_2^p} = \frac{X - I}{\left(\left(1 - \frac{1}{\alpha_1^p}\right) K(r - \mu_p^*) P^*_{A,p}(X) - I\right)} P^*_{A,p}(X)^{\alpha_2^p} \tag{35}$$

Which, combined with the previous expression, gives equation 2.5.1 implicitly characterising $P^*_{A,p}(X)$:

$$\frac{\left(\left(1 - \frac{1}{\alpha_2^p}\right) K(r - \mu_p^*) P^*_{A,p}(X) - I\right)^{\frac{1}{\alpha_1^p}}}{\left(\left(1 - \frac{1}{\alpha_1^p}\right) K(r - \mu_p^*) P^*_{A,p}(X) - I\right)^{\frac{1}{\alpha_2^p}}} = (X - I)^{\frac{1}{\alpha_1^p} - \frac{1}{\alpha_2^p}}$$

We then find $P^*_{A,p}(X)$ by solving numerically equation 2.5.1, and then derive $P^*_{BC,p}(X)$ using equation 18 or 35. For the consumer, the reasoning is similar : the frontiers can be defined by solving equation 17:

$$\frac{\left(U - \left(1 - \frac{1}{\alpha_2^c}\right) K(r - \mu_c^*) P^*_{A,c}(X)\right)^{\frac{1}{\alpha_1^c}}}{\left(U - \left(1 - \frac{1}{\alpha_1^c}\right) K(r - \mu_c^*) P^*_{A,c}(X)\right)^{\frac{1}{\alpha_2^c}}} = (U - X)^{\frac{1}{\alpha_1^c} - \frac{1}{\alpha_2^c}} \tag{36}$$

And then by using one of these two equations to determine the frontier to sign a PPA :

$$P^*_{BC,c}(X)^{\alpha_1^c} = \frac{U - X}{\left(U - \left(1 - \frac{1}{\alpha_2^c}\right) K(r - \mu_c^*) P^*_{A,c}(X)\right)} P^*_{A,c}(X)^{\alpha_1^c} \tag{37}$$

$$P^*_{BC,c}(X)^{\alpha_2^c} = \frac{U - X}{\left(U - \left(1 - \frac{1}{\alpha_1^c}\right) K(r - \mu_c^*) P^*_{A,c}(X)\right)} P^*_{A,c}(X)^{\alpha_2^c} \tag{38}$$

B Calibration Parameters & Sources

Consumer (ArcelorMittal)			
Benefits		**Costs**	
From Abatements	3933966412,13	Capital Expenditures	1850000000,00
Subsidies	850000000,00	Losses from production halt	398524004,25
From Sparred Fossil Fuels	426752066,71		
U (€, total)		2962194474,60	
LROEC		122,8904145	
U (€/MWh, Over Project Lifespan)		849,7402394	

Category	Amount	Source
Coal Emissions per MWh (t)	0,986	RTE
Gas Emissions per MWh (t)	0,43	RTE
Share of Gas (%)	10%	IEA Energy Statistics
Share of Coal (%)	90%	IEA Energy Statistics
Abatements (t)	70 000 000,00	European Commission
Sparred Coal (MWh)	67 720 090,29	Deduced
Sparred Gas (MWh)	7 524 454,48	Deduced
2030 future TTF Price (€/MWh)	26,91	LSEG / Refinitiv Workspace
2030 Coal Price (€/MWh)	13,04	LSEG / Refinitiv Workspace
Savings from fossil fuel	1 085 553 047,40	Deduced
CO2 Price	205,29	ADEME
Savings from Abatements	14 370 125 000,00	Deduced
Electricity consumption (MWh/t)	0,42	Ramezani Moziraji et al. [2023]
Steel production (t/y)	4 666 666,67	ArcelorMittal
Project lifespan (Y)	15,00	Deduced from ArcelorMittal
Total Energy consumption (MWh)	29 182 300,00	Deduced
WACC	10.2%	arc [2024]

	Market Data		
	Parameter	Value	Source
ArcelorMittal	μ_M^{prod}	0.0438	Eurostoxx 50 (LSEG / Refinitiv)
	σ_M^{prod}	0.1068	
	ρ_M^{prod}	-0.1985	
EDF	μ_M^{cons}	0.0621	MSCI Energy Utilities ETF (LSEG / Refinitiv)
	σ_M^{cons}	0.1683	
	ρ_M^{cons}	0.4210	
Electricity Markets	μ_P	0.0285	Non-Household Electricity Prices (Eurostat)
	σ_P	0.2175	
	$\hat{\eta}$	0.09	
Risk-free	r	2%	-

C Good Deal Bounds

In our theoretical model, all discount factors are endogenous, except the risk free rate. Thus, the discount factor r_C of any future risky cashflow C should be

$$r_C = r + \sigma_C \left(\rho_{SC} \times h_S + \sqrt{1 - \rho_{SC}^2} \times (\hat{\eta} + \kappa_i^*) \right) \tag{39}$$

With ρ_{SC} here representing the correlation between the risk source of the cashflow C and the market folio's risk source B_t. In the application case, we use discount factors provided by EDF and Arcelor Mittal's official documents of respectively 4% and 10%.

In the case of electricity however, this intuition does not directly apply to the asset P representing the electricity price because electricity cannot be stored for arbitrage. In our case, the bounds $[-\kappa_i, \kappa_i]$ are therefore derived indirectly from a similar, but longer term "limited-arbitrage" argument, involving generation and consumption capacities investment returns r^i:

$$r^p = \frac{NPV_A^p}{I} = -1 + \frac{1}{I}\mathbb{E}_t^{\mathcal{Q}_p} \left(\int_t^{t+T} e^{-r(\tau-t)} P_\tau \right) = -1 + \frac{K(r - \mu_p^*)}{I} P_t \tag{40}$$

$$r^c = \frac{NPV_A^c}{U} = 1 - \frac{1}{U}\mathbb{E}_t^{\mathcal{Q}_c} \left(\int_t^{t+T} e^{-r(\tau-t)} P_\tau \right) = 1 - \frac{K(r - \mu_c^*)}{U} P_t \tag{41}$$

With $K(\circ)$ the operator defined in equation 7.

The return r^p (resp. r^c) is decreasing (resp. increasing) in κ_i^* [18]. Imposing a maximal Good-Deal bound on the return r^p will therefore define a unique lower bound κ_{min} to the prior. Similarly,

[18] μ_i^* is a decreasing function in κ_i^* and K is a decreasing function in its argument.

imposing a maximal Good-Deal bound on the return r^c will define a unique higher bound κ_{max} to the prior.

The market being incomplete, these Good-Deal bounds are not shared among all market agents. They are rather a structural ambiguity aversion parameter for each agent, reflecting its own vision of the market. The segment of possible priors we just defined is therefore dependent on agent i : $[\kappa^i_{min}, \kappa^i_{max}]$.

D Deterministic case ($\sigma_P = 0$)

In the deterministic case, the volatility σ_P becomes zero (by definition), and thus the risk-neutral probability measure becomes the mere natural probability measure. The real option problem 16 can be reformulated as a deterministic optimal stopping problem :

$$F^i(P_t, X) = \max_{T\geq t} \Big(\max\,[NPV^i_A(P_T)\ ;\ NPV^i_{BC}(X)] e^{-r(T-t)} \Big) \tag{42}$$

$$= \max \Big[\max_{T\geq t} (NPV^i_A(P_T) e^{-r(T-t)})\ ;\ \max_{T\geq t} (NPV^i_{BC}(X) e^{-r(T-t)}) \Big] \tag{43}$$

$$= \max \Big[G^i(P_t)\ ;\ NPV^i_{BC}(X) \Big] \tag{44}$$

$$\tag{45}$$

Where $NPV^i_{BC} = X - I$ for the producer and $U - X$ for the consumer. $G^i(P_t)$ is the value defined by a deterministic optimal stopping problem which does not depend on X. These problems (one for the producer and one for the consumer) only depends on parameters I, U, μ and r. Their solutions can be found analytically following Dixit and Pindyck [1994] (p. 138) and depend on r and μ's values :

μ	**Producer**	**Consumer**
$+\infty$	$T^* = +\infty$ $G^p(P_t) = +\infty$	$T^* = t$ $G^c(P_t) = U - KP_t$
r	$T^* = t + \max\left\{0,\ \frac{1}{\mu}\log\left(\frac{rI}{(r-\mu)KP_t}\right)\right\}$ $P^* = \frac{r}{r-\mu}\cdot\frac{I}{K} > \frac{I}{K}$ $G^p(P_t) = \begin{cases} \frac{\mu I}{(r-\mu)K}\left(\frac{P_t}{P^*}\right)^{r/\mu} & P_t \leq P^* \\ KP_t - I, & P_t > P^* \end{cases}$	$T^* = t$ $G^c(P_t) = \max(U - KP_t,\ 0)$
0 $-\infty$	$T^* = t$ $G^p(P_t) = \max(KP_t - I,\ 0)$	$T^* = t + \max\left\{0,\ \frac{1}{\mu}\log\left(\frac{rU}{(r-\mu)KP_t}\right)\right\}$ $P^* = \frac{r}{r-\mu}\cdot\frac{U}{K} < \frac{U}{K}$ $G^c(P_t) = \begin{cases} \frac{\mu U}{(r-\mu)K}\left(\frac{P_t}{P^*}\right)^{r/\mu} & P_t \geq P^* \\ U - KP_t, & P_t < P^* \end{cases}$

If $\mu \geq r$ of course, no contract can be signed, because the investor will prefer to wait to invest alone whatever the strike price X. If $\mu < r$ however, there will be a zone of strike and market prices$\{X, P_t\}$ where the contract NPV will be greater than the option value : $NPV^i_{BC}(X) > G^i(P_t)$, and where the agent will be ready to sign a Bilateral Contract. Isolating X from the NPV expression, one gets the frontier values for each agent :

$$X^*_p(P_t) = I\ + G^p(P_t) \qquad \text{the producer signs the BC for } X \text{ above this threshold}$$
$$X^*_c(P_t) = U - G^c(P_t) \qquad \text{the consumer signs the BC for } X \text{ below this threshold}$$

By definition of G^p, one can see that $X^*_p(P_t) = KP_t$ when $P_t \geq P^*$ (in the real option case) or $P_t \geq I/K$ (in the NPV case). In all other cases, the frontier X^*_p will be strictly above the line KP_t. Similarly, for the consumer $X^*_c(P_t)$ will be equal to KP_t when P_t is either below P^* (in the real option case) or U/K (in the NPV case). In all other cases, the frontier X^*_c will be strictly below the line KP_t.

Finally, one can derive explicitly the contract zones for this zero-volatility case, depending on the parameter values :

- For $\mu < 0$: An agreement to sign a Bilateral Contract is possible when the market price P_t is in the interval $\left[\frac{I}{K}\ ,\ \frac{r}{r-\mu}\frac{U}{K}\right]$. If $\frac{I}{K} > \frac{r}{r-\mu}\frac{U}{K}$ then no agreement can be reached.

- For $0 < \mu < r$: An agreement to sign a Bilateral Contract is possible when the market price P_t is in the interval $\left[\frac{r}{r-\mu}\frac{I}{K}\ ,\ \frac{U}{K}\right]$. If $\frac{r}{r-\mu}\frac{I}{K} > \frac{U}{K}$ then no agreement can be reached.
- For $\mu > r$ an agreement can never be found.

In the examples presented here, the calibration in the theoretical part illustrates the first case with a decreasing drift for electricity prices, and a zone of possible agreement for low volatilities. In contrast, the case study calibrations (see Appendix 2.6) exhibit no possible agreement for low volatilities, because the narrow gap between I and U is quickly overlapped by the real option premium, even at low volatilities.

E Sensitivity analysis for the case study

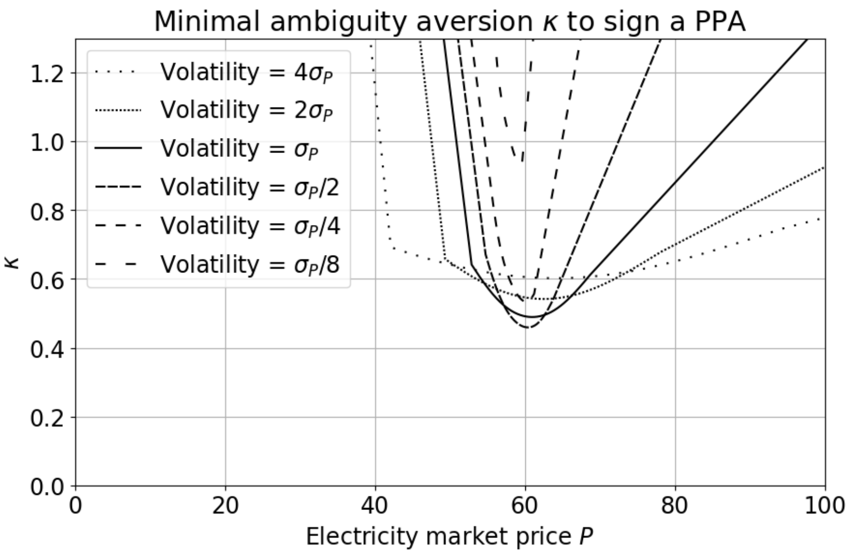


Figure 10: Minimum Ambiguity Levels to sign a contract

F Disclaimer

Use of AI: The authors have used the help of generative AI to help enhance and clarify some of the writing in this paper. The AI tools have specifically been used for language purposes, and have never been leveraged to generate new ideas or solve problems. The credit for that solely goes to the authors.

Task Repartition		
Task Completed	Jules Welgryn	Louis Soumoy
Original Idea	X	X
Literature Review	X	X
Model Design - Theory	X	Lead
Model Design - Case Study	Lead	X
Model Solving	X	X
Text and Structure	Lead	X
Code Writing	X	Lead
Presentations & Feedback	X	X

References

[1] ArcelorMittal 2024 Annual Report. Technical report, ArcelorMittal, 2024. URL https://corporate-cm-prod.arcelormittal.com/media/if0baqyg/annual-report-2024.pdf.

[2] Ibrahim Abada, Gauthier De Maere D'Aertrycke, Andreas Ehrenmann, and Yves Smeers. What Models Tell us about Long-term Contracts in Times of the Energy Transition. *Economics of Energy & Environmental Policy*, 8(1):163–182, 2019. ISSN 2160-5882. URL https://www.jstor.org/stable/27030649. Publisher: International Association for Energy Economics.

[3] ADEME. Cahier des Charges de l'Appel D'offres "Grands Projets Industriels de Décarbonation 2024". Technical report, ADEME, 2024.

[4] ArcelorMittal. Projet de production d'acier à basse émission de CO2 sur le site de Dunkerque. Technical report, ArcelorMittal, 2023.

[5] ArcelorMittal. Dunkerque : La perle du nord portée par l'industrie, March 2024. URL https://france.arcelormittal.com/activites/dunkerque-la-perle-du-nord-portee-par-l-industrie.

[6] Carlos Batlle, Tim Schittekatte, Paolo Mastropietro, and Pablo Rodilla. The EU Commission's Proposal for Improving the Electricity Market Design: Treading Water, but not Drowning. *Current Sustainable/Renewable Energy Reports*, 10(4):197–205, December 2023. ISSN 2196-3010. doi: 10.1007/s40518-023-00223-4. URL https://doi.org/10.1007/s40518-023-00223-4.

[7] Loïc Berger and Massimo Marinacci. Model Uncertainty in Climate Change Economics: A Review and Proposed Framework for Future Research. *Environmental and Resource Economics*, 77(3):475–501, November 2020. ISSN 1573-1502. doi: 10.1007/s10640-020-00503-3. URL https://doi.org/10.1007/s10640-020-00503-3.

[8] Hendrik Bessembinder and Michael L. Lemmon. Equilibrium pricing and optimal hedging in electricity forward markets. *The Journal of Finance*, 57(3):1347–1382, 2002. doi: https://doi.org/10.1111/1540-6261.00463. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/1540-6261.00463.

[9] Truman F. Bewley. Knightian decision theory. Part I. *Decisions in Economics and Finance*, 25(2):79–110, November 2002. ISSN 1129-6569. doi: 10.1007/s102030200006. URL https://doi.org/10.1007/s102030200006.

[10] Farhad Billimoria, Jacob Mays, and Rahmatallah Poudineh. Hedging and Tail Risk in Electricity Markets, April 2024. URL https://www.oxfordenergy.org/publications/hedging-and-tail-risk-in-electricity-markets/.

[11] Zorana Božić, Dušan Dobromirov, Jovana Arsić, Mladen Radišić, and Beata Ślusarczyk. Power Exchange Prices: Comparison of Volatility in European Markets. *Energies*, 13(21):5620, January 2020. ISSN 1996-1073. doi: 10.3390/en13215620. URL https://www.mdpi.com/1996-1073/13/21/5620. Number: 21 Publisher: Multidisciplinary Digital Publishing Institute.

[12] Serhan Cevik and Keitaro Ninomiya. Chasing the Sun and Catching the Wind: Energy Transition and Electricity Prices in Europe, November 2022.

[13] Zengjing Chen and Larry Epstein. Ambiguity, Risk, and Asset Returns in Continuous Time. *Econometrica*, 70(4):1403–1443, 2002. ISSN 0012-9682. URL https://www.jstor.org/stable/3082003. Publisher: [Wiley, Econometric Society].

[14] Cheng Cheng, Zhen Wang, Mingming Liu, Qiang Chen, Abimelech Paye Gbatu, and Xiaohang Ren. Defer option valuation and optimal investment timing of solar photovoltaic projects under different electricity market systems and support schemes. *Energy*, 127:594–610, May 2017. ISSN 0360-5442. doi: 10.1016/j.energy.2017.03.157. URL https://www.sciencedirect.com/science/article/pii/S0360544217305546.

[15] John Cochrane. *Asset Pricing, Revised Edition*. Princeton university press edition, January 2005. ISBN 978-0-691-12137-6. URL https://press.princeton.edu/books/hardcover/9780691121376/asset-pricing?srsltid=AfmBOor0Vkks_yS229OnCHFWlqGqGxZ6ButBKbqiorX3CxIZ3s6ka03M.

[16] Tine Compernolle, Peter M. Kort, and Jacco J. J. Thijssen. The effectiveness of carbon pricing: The role of diversification in a firm's investment decision. *Energy Economics*, 112:106115, August 2022. ISSN 0140-9883. doi: 10.1016/j.eneco.2022.106115. URL https://www.sciencedirect.com/science/article/pii/S0140988322002742.

[17] Gonzalo Cortazar, Eduardo S. Schwartz, and Marcelo Salinas. Evaluating Environmental Investments: A Real Options Approach. *Management Science*, 44(8):1059–1070, August 1998. ISSN 0025-1909. doi: 10.1287/mnsc.44.8.1059. URL https://pubsonline.informs.org/doi/abs/10.1287/mnsc.44.8.1059. Publisher: INFORMS.

[18] John C. Cox, Jonathan E. Ingersoll, and Stephen A. Ross. A Theory of the Term Structure of Interest Rates. *Econometrica*, 53(2):385–407, 1985. ISSN 0012-9682. doi: 10.2307/1911242. URL https://www.jstor.org/stable/1911242. Publisher: [Wiley, Econometric Society].

[19] Avinash K. Dixit and Robert S. Pindyck. *Investment Under Uncertainty*. Princeton University Press, 1994. ISBN 978-0-691-03410-2. Google-Books-ID: VahsELa_qC8C.

[20] Mario Draghi. The future of European competitiveness. Technical report, European Commission, 2024.

[21] EDF. Projet de construction d'une paire de réacteurs EPR2. Synthèse du dossier des maîtres d'ouvrage. Technical report, EDF, August 2024.

[22] Daniel Ellsberg. Risk, Ambiguity, and the Savage Axioms. *The Quarterly Journal of Economics*, 75(4):643–669, 1961. ISSN 0033-5533. doi: 10.2307/1884324. URL https://www.jstor.org/stable/1884324. Publisher: Oxford University Press.

[23] European Central Bank. *The interplay between green policy, electricity prices, financial constraints and jobs: firm level evidence.* Publications Office, LU, 2021. URL https://data.europa.eu/doi/10.2866/302949.

[24] European Commission. Aides d'État : la Commission autorise une mesure française d'un montant de 850 millions d'euros visant à aider ArcelorMittal à décarboner sa production d'acier - Commission européenne. URL https://france.representation.ec.europa.eu/informations/aides-detat-la-commission-autorise-une-mesure-francaise-dun-montant-de-850-millions-deuros-vi fr.

[25] European Environment Agency. European Environment Agency greenhouse gases — data viewer, April 2024. URL https://www.eea.europa.eu/en/analysis/maps-and-charts/greenhouse-gases-viewer-data-viewers.

[26] Foreign Ministry for Economic Affairs and Climate Action. Carbon Contracts for Difference - Klimaschutzverträge, 2023. URL https://www.klimaschutzvertraege.info/en/home.

[27] Itzhak Gilboa and David Schmeidler. Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics*, 18(2):141–153, January 1989. ISSN 0304-4068. doi: 10.1016/0304-4068(89)90018-9. URL https://www.sciencedirect.com/science/article/pii/0304406889900189.

[28] Nicholas Gohdes, Paul Simshauser, and Clevo Wilson. Renewable entry costs, project finance and the role of revenue quality in Australia's National Electricity Market. *Energy Economics*, 114:106312, October 2022. ISSN 0140-9883. doi: 10.1016/j.eneco.2022.106312. URL https://www.sciencedirect.com/science/article/pii/S0140988322004418.

[29] Krista Halttunen, Iain Staffell, Raphael Slade, Richard Green, Yves-Marie Saint-Drenan, and Malte Jansen. Global Assessment of the Merit-Order Effect and Revenue Cannibalisation for Variable Renewable Energy, December 2020. URL https://papers.ssrn.com/abstract=3741232.

[30] Geoffrey Heal and Antony Millner. Uncertainty and Decision in Climate Change Economics, March 2013. URL https://www.nber.org/papers/w18929.

[31] Vicky Henderson. Valuing the option to invest in an incomplete market. *Mathematics and Financial Economics*, 1(2):103–128, November 2007. ISSN 1862-9679, 1862-9660. doi: 10.1007/s11579-007-0005-z. URL http://link.springer.com/10.1007/s11579-007-0005-z.

[32] IEA Energy Statistics. Energy demand for iron and steel by fuel in the Net Zero Scenario, 2010-2030 – Charts – Data & Statistics. URL https://www.iea.org/data-and-statistics/charts/energy-demand-for-iron-and-steel-by-fuel-in-the-net-zero-scenario-2010-2030-2.

[33] Cosmin L. Ilut and Martin Schneider. Modeling Uncertainty as Ambiguity: a Review, April 2022. URL https://www.nber.org/papers/w29915.

[34] Karolina Kapral, Kobe Soetaert, and Rui Castro. An Off-Site Power Purchase Agreement (PPA) as a Tool to Protect against Electricity Price Spikes: Developing a Framework for Risk Assessment and Mitigation. *Energies*, 17(9):2161, January 2024. ISSN 1996-1073. doi: 10.3390/en17092161. URL https://www.mdpi.com/1996-1073/17/9/2161. Number: 9 Publisher: Multidisciplinary Digital Publishing Institute.

[35] Frank H. Knight. *Risk, Uncertainty and Profit*. Social Science Research Network, Rochester, NY, 1921. URL https://papers.ssrn.com/abstract=1496192.

[36] Sebastian Kolb, Marius Dillig, Thomas Plankenbühler, and Jürgen Karl. The impact of renewables on electricity prices in Germany - An update for the years 2014–2018. *Renewable and Sustainable Energy Reviews*, 134:110307, December 2020. ISSN 1364-0321. doi: 10.1016/j.rser.2020.110307. URL https://www.sciencedirect.com/science/article/pii/S1364032120305955.

[37] Zachary Lucy and Jordan Kern. Analysis of fixed volume swaps for hedging financial risk at large-scale wind projects. *Energy Economics*, 103:105603, November 2021. ISSN 0140-9883. doi: 10.1016/j.eneco.2021.105603. URL https://www.sciencedirect.com/science/article/pii/S0140988321004710.

[38] Michael Magill and Martine Quinzii. *Theory of Incomplete Markets*. Mit press edition, 1996. ISBN 978-0-262-13324-1.

[39] Harry Markowitz. Portfolio Selection. *The Journal of Finance*, 7(1):77–91, 1952. ISSN 0022-1082. doi: 10.2307/2975974. URL https://www.jstor.org/stable/2975974. Publisher: [American Finance Association, Wiley].

[40] Nils May and Karsten Neuhoff. Financing Power: Impacts of Energy Policies in Changing Regulatory Environments. *The Energy Journal*, 42(4):131–152, July 2021. ISSN 0195-6574. doi: 10.5547/01956574.42.4.nmay. URL https://doi.org/10.5547/01956574.42.4.nmay. Publisher: SAGE Publications.

[41] Jacob Mays, Michael T. Craig, Lynne Kiesling, Joshua C. Macey, Blake Shaffer, and Han Shu. Private risk and social resilience in liberalized electricity markets. *Joule*, 6(2):369–380, February 2022. ISSN 2542-4351. doi: 10.1016/j.joule.2022.01.004. URL https://www.sciencedirect.com/science/article/pii/S2542435122000368.

[42] Antony Millner, Simon Dietz, and Geoffrey Heal. Scientific Ambiguity and Climate Policy. *Environmental and Resource Economics*, 55(1):21–46, May 2013. ISSN 1573-1502. doi: 10.1007/s10640-012-9612-0. URL https://doi.org/10.1007/s10640-012-9612-0.

[43] David Newbery. Missing money and missing markets: Reliability, capacity auctions and interconnectors. *Energy Policy*, 94:401–410, July 2016. ISSN 0301-4215. doi: 10.1016/j.enpol.2015.10.028. URL https://www.sciencedirect.com/science/article/pii/S0301421515301555.

[44] Robert S Pindyck. THE LONG-RUN EVOLUTION OF ENERGY PRICES, 1999.

[45] Maziar Ramezani Moziraji, Ghorban Ali Dezvareh, Majid Ehteshami, Mohammad Reza Sabour, and Alireza Bazargan. Life cycle assessment of gas-based EAF steel production: environmental impacts and strategies for footprint reduction. *The International Journal of Life Cycle Assessment*, 28(12):1605–1621, December 2023. ISSN 1614-7502. doi: 10.1007/s11367-023-02230-5. URL https://link.springer.com/article/10.1007/s11367-023-02230-5. Company: Springer Distributor: Springer Institution: Springer Label: Springer Number: 12 Publisher: Springer Berlin Heidelberg.

[46] RTE. éCO2mix - Les émissions de CO2 par kWh produit en France | RTE. URL https://www.rte-france.com/eco2mix/les-emissions-de-co2-par-kwh-produit-en-france.

[47] Tim Schittekatte and Carlos Batlle. Power Crisis in the EU 3.0: Proposals to Complete Long-Term Markets, February 2023.

[48] Eduardo Schwartz and James E. Smith. Short-Term Variations and Long-Term Dynamics in Commodity Prices. *Management Science*, 46(7), July 2000. ISSN 1526-5507.

[49] Paul Simshauser. Missing money, missing policy and Resource Adequacy in Australia's National Electricity Market. *Utilities Policy*, 60:100936, October 2019. ISSN 0957-1787. doi: 10.1016/j.jup.2019.100936. URL https://www.sciencedirect.com/science/article/pii/S0957178718302947.

[50] Paul Simshauser, Yuan Tian, and Patrick Whish-Wilson. Vertical integration in energy-only electricity markets. *Economic Analysis and Policy*, 48(C):35–56, 2015. ISSN 0313-5926. URL https://econpapers.repec.org/article/eeeecanpo/v_3a48_3ay_3a2015_3ai_3ac_3ap_3a35-56.htm. Publisher: Elsevier.

[51] Peter Smith and Michael Wickens. Asset Pricing with Observable Stochastic Discount Factors. *Journal of Economic Surveys*, 16(3):397–446, 2002. ISSN 1467-6419. doi: 10.1111/1467-6419.00173. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/1467-6419.00173. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/1467-6419.00173.

[52] Shinichiro Takizawa and Atsuyuki Suzuki. Analysis of the decision to invest for constructing a nuclear power plant under regulation of electricity price. *Decision Support Systems*, 37(3): 449–456, June 2004. ISSN 0167-9236. doi: 10.1016/S0167-9236(03)00045-9. URL https://www.sciencedirect.com/science/article/pii/S0167923603000459.

[53] Jacco J. J. Thijssen. Incomplete markets, ambiguity, and irreversible investment. *Journal of Economic Dynamics and Control*, 35(6):909–921, June 2011. ISSN 0165-1889. doi: 10.1016/j.jedc.2010.12.001. URL https://www.sciencedirect.com/science/article/pii/S0165188910002575.

[54] Oldrich Vasicek. An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5(2):177–188, November 1977. ISSN 0304-405X. doi: 10.1016/0304-405X(77)90016-2. URL https://www.sciencedirect.com/science/article/pii/0304405X77900162.

[55] John von Neumann and Oskar Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, Princeton, 1st edition, 1944.