Sustainable Economy Research Group (S.E.R.G.)
@ CentraleSupélec Paris-Saclay University
Industrial Engineering Department (LGI)

https://www.serg.paris/

WORKING PAPER SERIES N°1

« Capacity Investment Decisions in Equilibrium: A Distributionally Robust Approach »

by

Ibrahim Abada and Julien Ancel

05-2025 WP

Capacity investment decisions in equilibrium: a distributionally robust approach*

Ibrahim Abada[†] & Julien Ancel[‡]

March 26, 2025

Abstract

In electricity systems, investment in generation capacity is subject to risk. The distribution of uncertain parameters on which investment decisions depend might not be fully observed in historical values. In Europe, this was recently illustrated by the crisis of exceptionally high power prices during the 2021-2023 period, which was subsequently followed by a regime of extremely low and even negative prices. In that vein, ambiguity aversion reflects a lack of confidence in the distribution of uncertainty, while risk aversion is concerned with realizations of uncertainty. We study a competitive market with investors who are averse to ambiguity. Such a market is represented as an equilibrium model, where each agent solves a Wasserstein distributionally robust optimization problem regarding its investment decisions. Investments could be hedged by contracts. We derive a convex reformulation of the problem, demonstrate the existence of equilibria, and prove a version of the welfare theorem in this ambiguous context. Via simulations, we find that, as with risk aversion, ambiguity aversion results in capacity-investment deferrals. We show however that, unlike standard results obtained with risk-aversion models, ambiguity cannot be hedged through financial contracts when their revenues are indexed on spot prices. Finally, we highlight that state-backed support schemes such as Contracts for Difference are welfare-improving and capacity-preserving under ambiguity.

Keywords- Distributionally robust optimization, Stochastic equilibrium, Power markets, Ambiguity, Risk.

---

*The opinions expressed in this paper are those of the authors alone. Email: julien.ancel@centralesupelec.fr

[†]Grenoble Ecole de Management, F-38000 Grenoble, France.

[‡]Industrial Engineering Research Department, CentraleSupelec, Paris Saclay University, France & Climate Economics Chair, Paris Dauphine University, France & Ecole Nationale des Ponts et Chaussées, France.

1 Introduction

1.1 Context: risk and uncertainty in power markets

Supported by regulatory schemes such as the Clean Energy Package and RePowerEU in the European Union and the Inflation Reduction Act in the United States, decarbonization and electrification policies will be closely intertwined in the coming decades. The power sector must undertake major investments in new low-carbon generation capacities (IEA [2023]). In this context, in liberalized power sectors, producers are expected to invest based on price signals from the energy market based on the crucial hypothesis that spot prices, i.e., short-term energy-only prices, carry valid long-term information. This assumption, which we owe to the seminal work of Marcel Boiteux (Boiteux [1960]), implies that long- and short-term marginal costs are equal in efficient markets provided that the production mix is adapted to demand. As highlighted notably in Joskow [2006], however, wholesale power markets are deemed imperfect, failing to bridge short- and long-term incentives owing to various market failures, among which are the exercise of market power and the well-known problem of 'missing money' (Joskow [2008]).

Risk and market agents' reactions to uncertainty constitute another fundamental market failure. In particular, the absence of sufficient risk-trading opportunities in the sole energy spot market has a detrimental effect on welfare when agents are risk-averse. This failure, referred to as market incompleteness or 'missing markets', has been highlighted in many articles in energy economics and operations research such as Finon [2008], Ehrenmann and Smeers [2011], Downward et al. [2012], Ralph and Smeers [2015], Newbery [2016], Philpott et al. [2016], and de Maere d Aertrycke et al. [2017], to cite but a few. This stream of research emphasizes the detrimental effects of market incompleteness on welfare and investments, advocating for the crucial role of contracts and other risk-sharing instruments in restoring welfare. This holds, of course, if the spot market is perfectly competitive, as some authors have demonstrated that contracts might fail to deliver on their promises in the presence of market power (Abada and Ehrenmann [2023]). From a modeling perspective, to the best of our knowledge the state-of-the-art approach to representing market incompleteness when investors are averse to risk utilizes multi-stage equilibrium models where risk aversion is captured via convex risk measures. Such measures provide, on the one hand, appreciable convexity properties, making it possible on the other hand to derive complementarity formulations and ease economic interpretation of the results via so-called *risk-adjusted* probabilities. These models have become standard when analyzing investment decisions in a risk-averse environment; recent articles include de Maere d Aertrycke and Smeers [2013], Egging and Holz [2016], Downward et al. [2016], Abada et al. [2017a], Ferris and Philpott [2022], Mays and Jenkins [2022], and Dimanchev et al. [2024].

This stream of research, and its global focus on distortions of investment incentives in the electricity sector by the perception of uncertainty, gained visibility in a series of recently published white papers documenting the recognition by regulators and policymakers of the serious issue of market incompleteness.[1] The issue has also gained practical reach as spot prices have, consecutively, starting in 2021, displayed an all-time high regime resulting from the Russo-Ukrainian war and gas disruptions, followed in 2024 by fast-paced growth in the number of hours with negative prices, as reported in Figure 1. In the span of a mere four years, European power systems have undergone two regimes at opposite price extremes. Figure 1 also indicates that these regimes deviate substantially from the standard distribution of power prices observed until 2020. Faced with such dramatic shifts in market data, could risk-averse investors still rely on historical data to value future revenues?

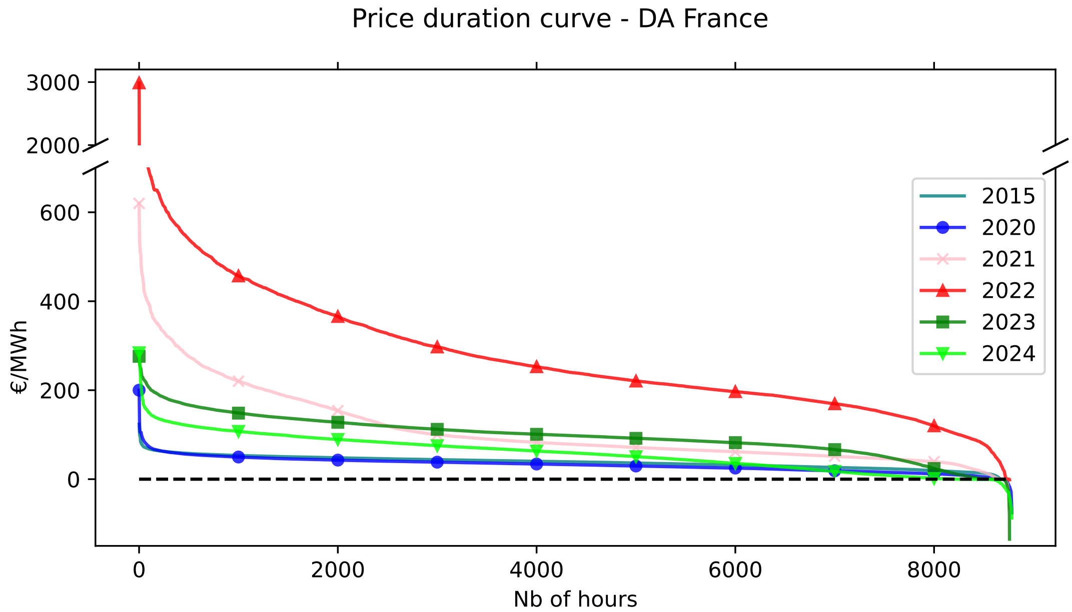

Figure 1: Day-ahead (DA) price-duration curves for France, in a selection of years between 2015 and 2024. Source: ENTSO-E, calculations by the authors.

1.2 The research questions

This question brings us back to the ontological difference between risk and uncertainty. In the abovementioned literature on risk, it is assumed that random variables (prices, demand, plants' availability, etc.) can be estimated from past data. In other words, while certain data may be unknown, historical observations offer sufficient information to infer their probability distributions *ex ante*. This approach may fall short, however, when past observations can no longer be relied upon, as argued above in the case of spot prices. Therefore, investors may no longer be

[1]Published by the Council of European Energy Regulators and the European Agency for the Cooperation of Energy Regulators, these papers discuss the issue of missing markets in Europe. See `https://www.ceer.eu/wp-content/uploads/2024/04/C21-FP-49-03_Paper-on-LT-investment-signals.pdf` and `https://www.acer.europa.eu/sites/default/files/documents/Publications/Final_Assessment_EU_Wholesale_Electricity_Market_Design.pdf`.

risk-averse in the traditional sense that they know the distribution of the random data they will face once an investment is made, but they may want their decisions to prove *robust* against an observed but not absolutely trustworthy distribution of the uncertain data instead. This novel feature of power markets, which we feel is adapted to the present situation, is referred to as investment *under ambiguity*, in contrast with standard models of investment *under risk*. Similarly, agents seeking optimal decisions in the context of an unknown distribution of data will be referred to as *ambiguity-averse*. Our research questions can be summarized as follows. i) How can we model a power economy where investment decisions have to be taken in an uncertain environment in which the distribution of observed data cannot be entirely relied upon? ii) What is the impact of aversion to ambiguity on welfare and installed capacity? iii) How does this impact compare with the effects of risk aversion?

To answer these questions, we turn to the framework of distributionally robust optimization (DRO hereafter). We refer the reader to Rahimian and Mehrotra [2019] for a review and simply cite here some recent applications of this field: Gao et al. [2018], Bertsimas et al. [2019], and Van Parys et al. [2021]. In a nutshell, distributionally robust optimization strives to formulate the worst-case expectation of random profits when the probability measure is drawn from a so-called *ambiguity set*. Such a set encompasses all distribution functions of some random parameters which are relatively close to the distribution drawn from historical observations of data. This framework has garnered considerable attention recently with advances in theoretical research seeking convex reformulations and tractable approximations of the DRO problem: examples include Goh and Sim [2010] and Wiesemann et al. [2014] to cite but a few. In close relation to our research, the seminal work of Mohajerin Esfahani and Kuhn [2018] finds a convex reformulation of the general problem under moderate assumptions that apply well in the context of power systems.

This article treats the case where the ambiguity set is defined via the Wasserstein metric. Recently, some researchers leveraged the DRO framework in the context of power markets: for instance, Pourahmadi and Kazempour [2021] model a moment-based distributionally robust capacity-expansion plan, Arrigo et al. [2022] focus on the optimal dispatch problem for reserve energy from the point of view of an ambiguity-aware benevolent planner, and Esteban-Pérez and Morales [2023] study the optimal power flow under ambiguity using a distributionally robust chance-constrained model. To the best of our knowledge, all studies tackling ambiguity in power systems in a distributionally robust framework consider a single optimizing agent (e.g., an investor in Pourahmadi and Kazempour [2021], a Transmission System Operator in Arrigo et al. [2022] and Esteban-Pérez and Morales [2023]). Our research follows suit and leverages some convex reformulations of Mohajerin Esfahani and Kuhn [2018], but diverges from (and, thereby, contributes to) the literature by considering a power economy of producers/investors and consumers interacting in the spot market and eventually signing contracts in an effort to hedge their revenues. This necessitates elaboration of stochastic *equilibrium* models of *ambiguity-averse* agents (we refer to Sun and Xu [2016] for some basic formulations and convergence results derived from the problem). After all, in liberalized power markets, spot-price realizations are, one the one hand, a consequence of market equilibrium between producers' decisions and demand levels, and, on the other hand, this market equilibrium is affected by the invested capacity via the merit-order effect. Furthermore, market agents may not face the same levels of risk or ambiguity aversion, which explains the need to extend the literature to study the economic equilibria of the power system under ambiguity.

In fact, DRO is not the only viable approach to modeling aversion to ambiguity, as economic theory offers interesting alternatives such as the maxmin expected utility framework (Gilboa and Schmeidler [1989]), the smooth ambiguity model (Klibanoff et al. [2005]), and the α-maxmin expected utility model (Klibanoff et al. [2022]). Compared with these approaches, the DRO framework yields several advantages: First, it does not require utility functions which might be difficult to elicit. Second, it makes it possible to incorporate financial risk-hedging instruments such as contracts and options in a straightforward manner. Finally, it provides valuable convexity properties that facilitate its integration into decision-making processes and optimization problems, allowing for the derivation of equilibrium formulations. All these reasons explain our choice to adopt the DRO framework to model ambiguity aversion.

1.3 Contributions and structure of the paper

This article offers three noteworthy contributions to the literature. The first is methodological as, to the best of our knowledge, our paper is the first to propose a model of the power economy with ambiguity-averse agents in equilibrium. We believe that this effort is crucial, inasmuch as market prices have recently followed patterns that were perhaps never observed before. Using the DRO paradigm with the Wasserstein norm in a classical two-stage stochastic decision process, we were able to derive a convex formulation of each market participant's objective under standard assumptions pertaining to investment and contracting. Importantly, the derived model is of the same order of complexity as standard models of risk-averse agents. We highlight conditions for the existence of equilibria and analyze the implications of contracts in the context of ambiguity. As a second contribution, we offer economic intuitions pertaining to the problem at hand by defining the concepts of ambiguity-adjusted profits and surplus, demonstrating that, under competitive bidding, the ambiguity-adjusted industry profit is zero. This finding grounds our proposal within the classical paradigm of perfectly competitive markets. We also discuss how the classical absence of arbitrage condition in the financial market adapts to the context of ambiguity-averse agents. Our third contribution is to undertake a stylized numerical application of our model to the French context, which is characterized by a mix of variable renewables and resources to be dispatched. In so doing, we were able to assess and quantify the impact of ambiguity aversion on installed capacity and welfare. We were also able to demonstrate that some contracts the revenues for which are uncertain—because, for instance, they depend on spot-market conditions—might add to ambiguity instead of resolving it. This implies that, contrary to the classical results in the literature that analyzes risk-averse agents, such contracts might not prove useful. On the other hand, futures contracts or Contracts for Differences do much better, as they remove the ambiguity pertaining to the spot-market price. For ease of exposition, all our results are systematically compared with the classical findings of the literature dedicated to assessing the impact of risk aversion on the power economy when the financial market is incomplete.

We view our work as an initial step in incorporating ambiguity with respect to the distribution of uncertain parameters into capacity-expansion models. To maintain clarity and focus, our models are deliberately simplified, leaving out certain technical complexities associated with the power sector. Nonetheless, we believe the models provide strong proof of concept, demonstrating the viability of our approach. The remainder of the paper is structured as follows. Section 2 first presents a lemma for reducing the $\mathcal{L}_1$ Wasserstein distributionally robust optimization problem with loss function as a parametric linear program with an affine objective. This lemma is then applied to the power-economy context to derive a complementarity formulation of the equilibrium for ambiguity-averse investors. This section also offers a proof of existence of equilibria. In Section 3 we provide several economic interpretations of our model and demonstrate the nullity of the ambiguity-adjusted power-industry profit with competitive pricing. In this section we also discuss the effects of contracts on ambiguity. Section 4 presents a numerical application of the power economy's investment equilibrium under risk and ambiguity aversion, showing the capacity-freezing incentive in an ambiguous context and the various hedging capabilities offered by long-term contracts. In Section 5 we summarize our work and present some policy recommendations.

2 Capacity-investment model under ambiguity

We consider the traditional two-stage stochastic problem of investment and the operation of generation capacity within the power sector, possibly with the exchange of financial contracts serving as real securities such as spot-indexed long-term contracts. The novelty of this work is that investors face an ambiguous wholesale electricity price, as motivated in the introduction. They make their first-stage decisions based on worst-case expectations of the second-stage outcomes, i.e. in a distributionally robust way. This section first presents a useful lemma for reducing this worst-case expectation to a convex program (Section 2.1). The economic equilibrium of the power sector economy with ambiguity-averse agents is then stated as a complementarity problem (Section 2.2).

2.1 Convex reduction of worst-case recourse cost under ambiguity

We formulate a corollary of Theorem 4.2 in Mohajerin Esfahani and Kuhn [2018], which is a generalization of their Corollary 5.4 and an adaptation to our framework. Hence, we consider the problem of the worst-case expected recourse cost in a two-stage stochastic program where the uncertain parameter is ambiguous, but the objective of the second stage depends in part on the realization of this parameter. Following their notations, let us have ξ as an ambiguous parameter which takes values in the convex and the closed set $\Xi = \{x \in \mathbb{R}^m; \underline{D} \leq x \leq \bar{D}\}$, $(\hat{\xi}_i)_{1\leq i\leq N}$ is a collection of past realizations of ξ and $\hat{\mathbb{P}}_N$, the associated empirical distribution of ξ, which is built from these very past observations. The recourse cost, i.e. the second-stage value of the two-stage stochastic program, is modeled via a loss function $l : \Xi \to \mathbb{R}$, where

$$l(\xi) = \inf_{y\in\mathbb{R}^u} \left\{ \langle Q^T y + \alpha, \xi\rangle; \text{ s.t. } Wy \geq h \right\} \tag{1}$$

is the value of a linear program with a feasible set assumed to be non-empty and compact. $\langle .,.\rangle$ denotes the dot product, $\alpha \in \mathbb{R}^m$, $Q^T \in \mathbb{R}^{m\times u}$, $W \in \mathbb{R}^{v\times u}$, and $h \in \mathbb{R}^v$, with u and v being integer numbers. As in Mohajerin Esfahani and Kuhn [2018], when moving to the first stage, we value the loss via the worst-case expectation over a Wasserstein ambiguity set. This set is defined as the ball centered on the empirical distribution $\hat{\mathbb{P}}_N$ and of given radius ε for the Wasserstein metric in the space of probability distributions, a ball that we denote by $\mathcal{B}_\varepsilon\left(\hat{\mathbb{P}}_N\right)$. We choose the $\mathcal{L}_1$ norm as the underlying norm on $\mathbb{R}^m$ for the Wasserstein metric. This choice is not constraining but facilitates the exposition inasmuch as it will make the convex reduction linear, as proved below. The worst-case expectation under ambiguity can be written as the following infinite-dimension optimization problem:

$$\sup_{\mathbb{Q}\in\mathcal{B}_\varepsilon\left(\hat{\mathbb{P}}_N\right)} \mathbb{E}_{\mathbb{Q}}\left[l(\xi)\right]. \tag{2}$$

We emphasize that this problem is a generalization of Corollary 5.4 in Mohajerin Esfahani and Kuhn [2018] as a result of the presence of parameter α in the objective of the second stage; they take this parameter as zero in their Corollary. This parameter α is, however, instrumental for our subsequent models as it will represent the payoff for financial contracts, which can belong to the set of ambiguous data. By a slight abuse of notation, if a vector $z_i \in \mathbb{R}^q$ is of dimension $q \in \mathbb{N}$ for some $i \in \{1, 2, ..., N\}$, we denote its coordinates by $z_{i1}, z_{i2}, ..., z_{iq}$.

Lemma 1 (Linear reduction). *The worst-case expectation Problem (2) reduces to the linear program*

$$\inf_{\substack{\lambda, s_i, y_{ik} \in \mathbb{R} \\ \gamma_i^1, \gamma_i^2 \geq 0}} \lambda \varepsilon + \frac{1}{N} \sum_{i=1}^{N} s_i \tag{3a}$$

$$\text{s.t.} \quad W y_i \geq h, \qquad \forall i \in \{1, \ldots, N\} \tag{3b}$$

$$-\lambda \leq Q^T y_{ik} + \alpha_k - \gamma_{ik}^1 + \gamma_{ik}^2 \leq \lambda, \qquad \forall i \in \{1, \ldots, N\}, k \in \{1, \ldots, m\} \tag{3c}$$

$$\langle Q^T y_i + \alpha - \gamma_i^1 + \gamma_i^2, \hat{\xi}_i \rangle + \langle \gamma_i^1, \bar{D} \rangle - \langle \gamma_i^2, \underline{D} \rangle \leq s_i, \qquad \forall i \in \{1, \ldots, N\}. \tag{3d}$$

The proof is provided in Appendix B.1. This extension of the results reported in Mohajerin Esfahani and Kuhn [2018] supports our subsequent analysis of investment incentives under ambiguity for controllable generation technologies.

At this stage in the development of our models, we make a fundamental observation. In the context of an investment problem, a producer's second-stage objective, as modeled in relationship (1), represents its stochastic operational cost in each realization i of the second stage. Therefore, given the expression of the feasibility set of optimization Problem (1), we observe that the latter does not contain stochastic parameters. In other words, uncertain data are contained in the objective function only. This, unfortunately, limits the applicability of this approach to renewable assets (wind, solar . . .) as they would require accounting for the uncertainty stemming from spot prices—in the objective function—and from the load factor of the assets—in the constraints. Another result from Mohajerin Esfahani and Kuhn [2018] provides a convex reformulation of the worst-case recourse cost when the ambiguous parameters appear *only* in the constraints of the second-stage problem, limiting its usefulness in our setting. Unfortunately, despite our best efforts, we could not provide a convex reformulation for the case where ambiguity is present both in the objective *and* the constraints of the recourse-cost problem. Therefore, in the remainder of this article, we focus on the effects of ambiguity on investments by traditional generators and consider that wind and solar production is exogenous by reasoning on the residual load, i.e. demand net of renewable production. Methodologically, this feature is a clear shortcoming of the DRO paradigm with respect to risk aversion.

2.2 The agents of the power economy under ambiguity

The power sector economy comprises a perfectly competitive spot market and a set $\mathcal{C}$ of financial instruments for trading risk between market agents (here, essentially for trading spot-indexed long-term contracts). As alluded to above, we model a two-stage decision process. Investment and contracting decisions are taken in the first stage and physical trading of electricity, which we model along with the optimal operation of power plants, occurs in the second stage. The second stage is stochastic in the sense that demand, market prices, and payoffs on contracts are unknown in the first stage. We model the general case where payoffs on contracts can be indexed on spot prices or other market data. Market agents have access to historical data on these random variables, allowing them to build a dataset of N realizations, which we index by $i \in \{1, ..., N\} = \mathcal{N}$. Agents of the economy define a set $\mathcal{G}$ of generators, each of which possesses, for simplicity, a unique production technology (e.g., CCGT, nuclear . . .) and a representative consumer, which controls load curtailment. The operation phase, modeled by the second stage, comprises representative timeblocks $t \in \{1, \ldots, T\} = \mathcal{T}$, with respective durations of H_t hours. We might also refer to timeblocks as periods. All agents are price-takers and know the structure of the spot and financial markets. They all have access to the same sample of N realizations of the residual demand, which we denote by $(D_{it})_{1\leq i\leq N}$ in each period t. Each contract $c \in \mathcal{C}$ yields revenue $(p^2_{ic})_{1\leq i\leq N}$ for realization i and costs p^1_c in the first stage. As noted above, we consider the general formulation where contracts' returns p^2_{ic} are stochastic but this is not constraining as it suffices to make this parameter constant to account for futures or forward contracts, as developed in Section 3.3.2. As in classical models of contracting under risk, the prices of contracts p^1_c are endogenous to our model. Agents are ambiguity-averse towards uncertainty. Furthermore, the formation of the spot price is the outcome of a market-clearing process, which we can model by an equilibrium problem for each realization i. The corresponding market price at time t is denoted by $(p_{it})_{1\leq i\leq N}$.

The general philosophy we pursue is the following. Every agent knows that, in the second stage, there is a market-clearing process on the physical spot market (electricity exchanges) and the financial market (contracts' payoffs exchanges) which provide market prices and optimal dispatch in each possible realization i. This process delivers some profit to the agent. This distribution of profit, which is built based on past data, is then valued in the first stage in an ambiguity-averse way when each agent calculates optimal investments and contracting strategies. For the sake of clarity, we reiterate that all agents consider the same data for realizations $i \in \{1, 2, ...N\}$ given that they all have access to the same market information. This assumption ensures consistency as all market agents will anticipate the same market prices and contract payoffs in each realization i when calculating their first-stage decisions.

2.2.1 The generators

Consider generator $g \in \mathcal{G}$, who can invest in generation capacity K_g and sign contracts whose volumes are denoted by $(W_{gc})_{c\in\mathcal{C}}$. A positive volume means that the contract is bought; otherwise, it is sold. The marginal capital cost is assumed to be constant C_g. In the second stage, the generator maximizes its profit by targeting a volume of generation in each period x_{igt}, given the installed capacity, financial investments, and realization i. The generator's marginal generation cost c_{igt} is constant and random.

The second-stage problem for the generator g when scenario i is realized for uncertain parameters can be written as (the generator minimizes its cost for each realization):

$$l_g(K_g, W_g, \xi_{ig}) = \min_{x_{igt} \geq 0} \sum_t H_t(c_{igt} - p_{it})x_{igt} + \sum_c (p_c^1 - p_{ic}^2)W_{gc} \tag{4}$$
$$\text{s.t. } \forall t \in \mathcal{T}, \qquad x_{igt} \leq K_g \qquad [\mu_{igt}],$$

with dual variables written in front of their associated constraints. This problem is equivalent to solving for the necessary and sufficient KKT conditions

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \qquad 0 \leq \mu_{igt} \perp x_{igt} - K_g \leq 0 \tag{5a}$$
$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \qquad 0 \leq x_{igt} \perp H_t\left(p_{it} - c_{igt}\right) - \mu_{igt} \leq 0. \tag{5b}$$

In particular, variables μ_{igt} represent second-stage scarcity rents. At the investment stage, generator g minimizes its investment cost plus its worst-case expected loss (or cost) incurred in the operation phase. We hold that the uncertain parameter ξ_g from which ambiguity stems comprises the (opposite of) inframarginal rents the generator captures in the spot market and the (opposite of) the return on the financial contracts:

$$\forall i \in \mathcal{N}, \qquad \xi_{ig} = \begin{pmatrix} c_{igt} - p_{it} \\ p_c^1 - p_{ic}^2 \end{pmatrix} \in \mathbb{R}^{T+|\mathcal{C}|}. \tag{6}$$

As explained above, generator g observes the empirical distribution $\hat{\mathbb{P}}_g$ derived from historical observations of random parameter ξ_g but is sensitive to ambiguity. We model generator g's ambiguity aversion by computing its worst-case expectation over the ball of radius ε_g centered on the empirical distribution $\hat{\mathbb{P}}_g$ for the $\mathcal{L}_1$ Wasserstein metric. We build support for this uncertain parameter by assuming that it is bounded in the eye of generator g:

$$\begin{pmatrix} \underline{A}_g \\ \underline{B}_g \end{pmatrix} \leq \xi_g \leq \begin{pmatrix} \bar{A}_g \\ \bar{B}_g \end{pmatrix}, \tag{7}$$

with $\underline{A}_g$ and $\overline{A}_g$ belonging to $\mathbb{R}^T$, and $\underline{B}_g$ and $\overline{B}_g$ belonging to $\mathbb{R}^{|\mathcal{C}|}$. The existence of this support is, in fact, quite natural, as market prices are bounded by price caps, given that consumers have a bounded willingness-to-pay for electricity. Furthermore, contract prices are also bounded as they are either fixed or indexed on spot prices. At this stage in the development of our models, we mention the fundamental remark that, because the spot and financial markets clear in equilibrium, the realizations of the uncertain parameter ξ_g faced by generator g depend on first-stage decisions taken by all agents (invested capacities in particular) via the merit-order effect. Therefore, we sometimes write this uncertain parameter as $\xi_g(\boldsymbol{K}, \boldsymbol{W})$, where, for ease of exposition, we concatenate all investment decision variables into a vector $\boldsymbol{K}$ and all contracting decisions into a vector $\boldsymbol{W}$.

We can now state the first-stage problem in generator g's first-stage problem as

$$\inf_{\substack{K_g \geq 0 \\ W_g \in \mathbb{R}^{|\mathcal{C}|}}} C_g K_g + \sup_{\mathbb{Q} \in \mathcal{B}_{\varepsilon_g}(\hat{\mathbb{P}}_g)} \mathbb{E}_{\mathbb{Q}} \left[l_g \Big(K_g, W_g, \xi_g(\boldsymbol{K}, \boldsymbol{W}) \Big) \right]. \tag{8}$$

It is important to remark here that optimization Problem (8) does not involve second-stage decision variables x_{igt} or μ_{igt} calculated from (5a) and (5b), but only market prices p_{it} and contract payoffs p^2_{ic}. These variables are outcomes of a clearing process that we write explicitly in Section 2.2.3 and which involves these second-stage equations and variables.

Using Lemma 1, Problem (8) reduces to the following linear program (hereafter, for ease of notation, we might not write the bounds of some summation signs and consider, implicitly, that $i \in \{1, ..., N\}$, $c \in \mathcal{C}$, and $t \in \{1, ..., T\}$),

$$\inf_{\substack{K_g, x'_{igt}, \gamma^1_{igt}, \gamma^2_{igt}, \gamma^1_{igc}, \gamma^2_{igc} \geq 0 \\ \lambda_g, s_{ig}, W_{gc} \in \mathbb{R}}} C_g K_g + \lambda_g \varepsilon_g + \frac{1}{N} \sum_i s_{ig} \tag{9}$$

$$\begin{aligned}
\text{s.t.} \quad & \forall i \in \mathcal{N}, t \in \mathcal{T}, \quad x'_{igt} \leq K_g && [\mu'_{igt}] \\
& \forall i \in \mathcal{N}, \quad \left\{ \begin{aligned} & \textstyle\sum_t (H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt})(c_{igt} - p_{it}) + \gamma^1_{igt} \bar{A}_g - \gamma^2_{igt} \underline{A}_g \\ & + \textstyle\sum_c (W_{gc} - \gamma^1_{igc} + \gamma^2_{igc})(p^1_c - p^2_{ic}) + \gamma^1_{igc} \bar{B}_g - \gamma^2_{igc} \underline{B}_g \end{aligned} \right. \leq s_{ig} && [\alpha_{ig}] \\
& \forall i \in \mathcal{N}, t \in \mathcal{T}, \quad -\lambda_g \leq H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt} \leq \lambda_g && [\beta^2_{igt}, \beta^1_{igt}] \\
& \forall c \in \mathcal{C}, \quad -\lambda_g \leq W_{gc} - \gamma^1_{igc} + \gamma^2_{igc} \leq \lambda_g && [\beta^2_{igc}, \beta^1_{igc}].
\end{aligned}$$

Variables x'_{igt} model how producer g anticipates, in the first stage, its second-stage production in a distributionally robust way. Therefore, they generally differ from variables x_{igt} involved in Problem (4), which model the true production at realization i. Similarly, dual variable μ'_{igt} model the ambiguity-adjusted scarcity rents, which might differ from variables μ_{igt}.

Problem (9) is equivalent to the following KKT conditions:

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \le \mu'_{igt} \perp x'_{igt} - K_g \le 0 \tag{10a}$$

$$\forall i \in \mathcal{N}, \quad 0 \ge \alpha_{ig} \perp \begin{cases} \sum_t (H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt})(c_{igt} - p_{it}) + \gamma^1_{igt}\bar{A}_g - \gamma^2_{igt}\underline{A}_g \\ + \sum_c (W_{gc} - \gamma^1_{igc} + \gamma^2_{igc})(p^1_c - p^2_{ic}) + \gamma^1_{igc}\bar{B}_g - \gamma^2_{igc}\underline{B}_g - s_{ig} \end{cases} \le 0 \tag{10b}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \ge \beta^1_{igt} \perp H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt} - \lambda_g \le 0 \tag{10c}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \ge \beta^2_{igt} \perp -H_t x'_{igt} + \gamma^1_{igt} - \gamma^2_{igt} - \lambda_g \le 0 \tag{10d}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \ge \beta^1_{igc} \perp W_{gc} - \gamma^1_{igc} + \gamma^2_{igc} - \lambda_g \le 0 \tag{10e}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \ge \beta^2_{igc} \perp -W_{gc} + \gamma^1_{igc} - \gamma^2_{igc} - \lambda_g \le 0 \tag{10f}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \le x'_{igt} \perp -\mu'_{igt} - H_t\beta^2_{igt} + H_t\beta^1_{igt} + \alpha_{ig}H_t(c_{igt} - p_{it}) \le 0 \tag{10g}$$

$$0 \le K_g \perp -C_g + \sum_i \sum_t \mu'_{igt} \le 0 \tag{10h}$$

$$\forall c \in \mathcal{C}, \quad W_{gc} \perp \sum_i \left[\alpha_{ig}(p^1_c - p^2_{ic}) + \beta^1_{igc} - \beta^2_{igc}\right] = 0 \tag{10i}$$

$$\lambda_g \perp -\varepsilon_g - \sum_t \sum_i (\beta^1_{igt} + \beta^2_{igt}) - \sum_c \sum_i (\beta^1_{igc} + \beta^2_{igc}) = 0 \tag{10j}$$

$$\forall i \in \mathcal{N}, \quad s_{ig} \perp -\frac{1}{N} - \alpha_{ig} = 0 \tag{10k}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \le \gamma^1_{igt} \perp \beta^2_{igt} - \beta^1_{igt} - \alpha_{ig}(c_{igt} - p_{it}) + \alpha_{ig}\bar{A}_g \le 0 \tag{10l}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \le \gamma^2_{igt} \perp -\beta^2_{igt} + \beta^1_{igt} + \alpha_{ig}(c_{igt} - p_{it}) - \alpha_{ig}\underline{A}_g \le 0 \tag{10m}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \le \gamma^1_{igc} \perp \beta^2_{igc} - \beta^1_{igc} - \alpha_{ig}(p^1_c - p^2_{ic}) + \alpha_{ig}\bar{B}_g \le 0 \tag{10n}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \le \gamma^2_{igc} \perp -\beta^2_{igc} + \beta^1_{igc} + \alpha_{ig}(p^1_c - p^2_{ic}) - \alpha_{ig}\underline{B}_g \le 0. \tag{10o}$$

We draw the attention of the reader to the fact that, when $\varepsilon_g = 0$, it can be shown that the producer becomes risk-neutral and values all realizations i with equal probability $\frac{1}{N}$. On the other hand, when $\varepsilon_g \to +\infty$, the producer adopts robust behavior whereby only the worst realization of the second-stage cost, considering the support of ξ_g provided in (7), is accounted for when calculating the investment and contract volumes.

Combining equations (10i) and (10k) gives:

$$p^1_c = \frac{\sum_{i=1}^N p^2_{ic}}{N} + \frac{\sum_{i=1}^N \left[\beta^1_{igc} - \beta^2_{igc}\right]}{N} \quad \forall c \in \mathcal{C}. \tag{11}$$

This implies that the price of contract c is equal to the expectation of its second-stage revenue plus a premium measured by $\frac{\sum_i [\beta^1_{igc} - \beta^2_{igc}]}{N}$, which accounts for the fact that producer g is robust with respect to the ambiguity pertaining to the realization of prices p^2_{ic}. In particular, it can be shown that, when agent g is neutral to risk, i.e., $\varepsilon_g = 0$, the premium is equal to zero. This condition generalizes, conceptually, the standard *absence of arbitrage* condition of the financial market between the second and first stages in asset- pricing theory (Cochrane [2009]) to the case of ambiguity-averse market agents. Similarly, combining (10g) with (10h) gives the following investment criterion if $K_g > 0$:

$$C_g = \sum_{i=1}^{N} \sum_{t \in \mathcal{T}} \mu'_{igt} \tag{12}$$

$$= \frac{1}{N} \sum_{i=1}^{N} \sum_{t \in \mathcal{T}} H_t \left(p_{it} - c_{igt}\right) + \sum_{i=1}^{N} \sum_{t \in \mathcal{T}} H_t \left(\beta^1_{igt} - \beta^2_{igt}\right). \tag{13}$$

We interpret this result by stating that, according to relationship (13), the producer invests if only it trusts that it recoups its investment cost via the expectation of second-stage short-term margins $\left(p_{it} - c_{igt}\right)$, which it adjusts by a premium $\sum_{i=1}^{N} \sum_{t \in \mathcal{T}} H_t \left(\beta^1_{igt} - \beta^2_{igt}\right)$ to account for ambiguity of the distribution of these margins. Another interpretation can be derived from relationship (12), stating that the investment is undertaken if the capital cost can be recovered through the ambiguity-adjusted scarcity rents μ'_{igt}.

2.2.2 The consumer

As in the case of the generators, we consider a representative consumer whose objective is to maximize the surplus accrued from electricity consumption. The consumer values electricity at level PC (which could represent its willingness to pay, or the classical Value of Lost Load, VoLL in electricity markets) and can curtail the load. We model curtailment by variable e_{it} for realization i and period t. Therefore, in period t and at realization i of the uncertain parameters, the consumer's surplus is $(PC - p_{it})\,(D_{it} - e_{it})\,H_t$. Moreover, the consumer also engages in financial contracts W_{dc}, the return on which manifests in the second stage. In realization i, the second-stage problem of the consumer can be written as

$$l_d(W_d, \xi_{id}) = \min_{e_{it} \geq 0} \sum_t H_t(e_{it} - D_{it})(PC - p_{it}) + \sum_c W_{dc}(p^1_c - p^2_{ic}), \tag{14}$$

which translates to the equivalent KKT conditions

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \qquad 0 \leq e_{it} \perp p_{it} - PC \leq 0. \tag{15}$$

At the contracting stage, the consumer is averse to ambiguity. Here again, we concatenate the ambiguity parameter into

$$\forall i \in \mathcal{N}, \qquad \xi_{id} = \begin{pmatrix} PC - p_{it} \\ p^1_c - p^2_{ic} \end{pmatrix} \in \mathbb{R}^{T+|\mathcal{C}|}, \tag{16}$$

and build support for it as follows:

$$\begin{pmatrix} \underline{A}_d \\ \underline{B}_d \end{pmatrix} \leq \xi_d \leq \begin{pmatrix} \bar{A}_d \\ \bar{B}_d \end{pmatrix}. \tag{17}$$

Given a level of ambiguity aversion denoted by the radius ε_d of the $\mathcal{L}_1$ Wasserstein ball centered around the empirical distribution of ξ_d, which we denote by $\hat{\mathbb{P}}_d$, the representative consumer signs financial contracts to minimize the worst-case expectation of the second-stage cost (or loss of surplus). The first-stage problem of the consumer hence can be written as

$$\inf_{W_d \in \mathbb{R}^{|\mathcal{C}|}} \sup_{\mathbb{Q} \in \mathcal{B}_{\varepsilon_d}(\hat{\mathbb{P}}_d)} \mathbb{E}_{\mathbb{Q}} \Big[l_d \Big(W_d, \xi_d(\boldsymbol{K}, \boldsymbol{W}) \Big) \Big]. \tag{18}$$

According to Lemma 1, this problem reduces again to the following linear program:

$$\inf_{\substack{W_{dc}, \lambda_d, s_{id} \in \mathbb{R} \\ e'_{it}, \gamma^1_{idt}, \gamma^2_{idt}, \gamma^1_{idc}, \gamma^2_{idc} \geq 0}} \quad \lambda_d \varepsilon_d + \frac{1}{N} \sum_i s_{id} \tag{19}$$

$$\forall i \in \mathcal{N}, \quad \begin{cases} \sum_t [H_t(e'_{it} - D_{it}) - \gamma^1_{idt} + \gamma^2_{idt}](PC - p_{it}) + \gamma^1_{idt} \bar{A}_d - \gamma^2_{idt} \underline{A}_d \\ + \sum_c [W_{dc} - \gamma^1_{idc} + \gamma^2_{idc}](p^1_c - p^2_{ic}) + \gamma^1_{idc} \bar{B}_d - \gamma^2_{idc} \underline{B}_d \end{cases} \leq s_{id} \qquad [\alpha_{id}]$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad -\lambda_d \leq H_t(e'_{it} - D_{it}) - \gamma^1_{idt} + \gamma^2_{idt} \leq \lambda_d \qquad [\beta^2_{idt}, \beta^1_{idt}]$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad -\lambda_d \leq W_{dc} - \gamma^1_{idc} + \gamma^2_{idc} \leq \lambda_d \qquad [\beta^2_{idc}, \beta^1_{idc}],$$

which is equivalent to the following KKT conditions

$$\forall i \in \mathcal{N}, \quad 0 \geq \alpha_{id} \perp \begin{cases} \sum_t [H_t(e'_{it} - D_{it}) - \gamma^1_{idt} + \gamma^2_{idt}](PC - p_{it}) + \gamma^1_{idt}\bar{A}_d - \gamma^2_{idt}\underline{A}_d \\ + \sum_c [W_{dc} - \gamma^1_{idc} + \gamma^2_{idc}](p^1_c - p^2_{ic}) + \gamma^1_{idc}\bar{B}_d - \gamma^2_{idc}\underline{B}_d \end{cases} - s_{id} \leq 0 \tag{20a}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \geq \beta^1_{idt} \perp H_t(e'_{it} - D_{it}) - \gamma^1_{idt} + \gamma^2_{idt} - \lambda_d \leq 0 \tag{20b}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \geq \beta^2_{idt} \perp -H_t(e'_{it} - D_{it}) + \gamma^1_{idt} - \gamma^2_{idt} - \lambda_d \leq 0 \tag{20c}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \geq \beta^1_{idc} \perp W_{dc} - \gamma^1_{idc} + \gamma^2_{idc} - \lambda_d \leq 0 \tag{20d}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \geq \beta^2_{idc} \perp -W_{dc} + \gamma^1_{idc} - \gamma^2_{idc} - \lambda_d \leq 0 \tag{20e}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \leq e'_{it} \perp \alpha_{id}(PC - p_{it}) - \beta^2_{idt} + \beta^1_{idt} \leq 0 \tag{20f}$$

$$\forall c \in \mathcal{C}, \quad W_{dc} \perp \sum_i \left[\alpha_{id}(p^1_c - p^2_{ic}) + \beta^1_{idc} - \beta^2_{idc}\right] = 0 \tag{20g}$$

$$\lambda_d \perp -\varepsilon_d - \sum_t \sum_i (\beta^1_{idt} + \beta^2_{idt}) - \sum_c \sum_i (\beta^1_{idc} + \beta^2_{idc}) = 0 \tag{20h}$$

$$\forall i \in \mathcal{N}, \quad s_{id} \perp -\frac{1}{N} - \alpha_{id} = 0 \tag{20i}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \geq \gamma^1_{idt} \perp \beta^2_{idt} - \beta^1_{idt} - \alpha_{id}(PC - p_{it}) + \alpha_{id}\bar{A}_d \leq 0 \tag{20j}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \geq \gamma^2_{idt} \perp -\beta^2_{idt} + \beta^1_{idt} + \alpha_{id}(PC - p_{it}) - \alpha_{id}\underline{A}_d \leq 0 \tag{20k}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \geq \gamma^1_{idc} \perp \beta^2_{idc} - \beta^1_{idc} - \alpha_{id}(p^1_c - p^2_{ic}) + \alpha_{id}\bar{B}_d \leq 0 \tag{20l}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \geq \gamma^2_{idc} \perp -\beta^2_{idc} + \beta^1_{idc} + \alpha_{id}(p^1_c - p^2_{ic}) - \alpha_{id}\underline{B}_d \leq 0. \tag{20m}$$

2.2.3 The Nash equilibrium, existence result, and economic interpretation

The economic equilibrium of our power economy is modeled by simultaneously solving the KKT conditions for all agents: (10a) to (10o) for all generators $g \in \mathcal{G}$, and (20a) to (20m) for the consumer. Spot and contract prices (when they are indexed on spot-market data) are obtained by solving the second-stage equilibrium problem composed of relationships (5a)-(5b) and (15). We add the following conditions for spot and financial market-clearing:

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad p_{it} \perp D_{it} - \sum_g x_{igt} - e_{it} = 0 \tag{21a}$$

$$\forall c \in \mathcal{C}, \quad p^1_c \perp W_{dc} + \sum_g W_{gc} = 0. \tag{21b}$$

Equation (21a) defines the realization of spot prices. As explained above, all realizations are assumed to be the same for all agents. This assumption might seem strong but it is similar in spirit to the one adopted in all studies modeling investment decisions under risk aversion, which holds that the set of scenarios representing randomness are the same for all market agents and that these agents all have the same information about market fundamentals in every scenario (see for instance Ralph and Smeers [2011], Ehrenmann and Smeers [2011], Abada et al.

[2017b], Philpott et al. [2016], de Maere d Aertrycke and Smeers [2013], and similar studies).

Definition 1. *In the remainder of the present paper, we denote by P our equilibrium problem as constituted by equations (10a)-(10o) for all generators $g \in \mathcal{G}$, (20a)-(20m) for the consumer, market-clearing constraints (21a)-(21b), and the second-stage equilibrium problem composed of equations (5a)-(5b) and (15).*

Every agent solves a convex optimization problem such that its equivalent KKT conditions form a monotonous complementarity problem, given market-price realizations and contract prices. Yet, the economic equilibrium, which is formed by concatenating all KKT and clearing conditions, is not necessarily a monotonous complementarity problem. The existence of a solution to this equilibrium can be shown using a fixed-point argument provided that all variables belong to a compact set. It can be shown that physical variables (second-stage production, curtailment, and invested capacity) are bounded by a limit set by the demand levels. Realizations of the spot price are non-negative and bounded by the VoLL. Variables α_{ig} are bounded because of (10k) (and similarly for the consumer). Variables β^1_{igt}, β^2_{igt}, β^1_{igc}, and β^2_{igc} are bounded because of (10j) (and similarly for the consumer). Variables λ_g are bounded because, if they are not for say producer g, then the optimization problem (9) is unbounded (the objective being equal to $+\infty$), which is absurd because a producer always has the choice not to invest, which would yield a finite objective (similar reasoning applies to the consumer). This, in turn, will imply that variables γ^1_{igt}, γ^2_{igt} γ^1_{igc}, and γ^2_{igc} are bounded. To demonstrate this claim, one can observe that γ^1_{igt} and γ^2_{igt} cannot both be positive at the same time for any tuple (i, g, t) because of (10l) and (10m). Therefore, thanks to (10c) and (10d) and the boundedness of λ_g, one can deduce that γ^1_{igt} and γ^2_{igt} are bounded. Similar logic applies for variables γ^1_{igc} and γ^2_{igc}, provided that contract positions W_{gc} are bounded, which we demonstrate below. This reasoning naturally translates to the consumer. It can also be verified that contract prices are bounded because of the absence of arbitrage conditions in the form (11). It remains to verify the point that contract volumes are bounded, which is the most difficult task in the proof. Fortunately, a similar effort has already been undertaken in the literature that studies equilibrium problems under risk aversion and market incompleteness (see for instance de Maere d'Aertrycke and Smeers [2013]), providing a valuable source of inspiration for our work. In that literature, existence is ensured on the assumption that the interior of the intersection of the risk sets of all agents in the economy is not empty. In the present paper, we require a similar assumption, which we now present.

We remind readers that $\boldsymbol{K}$ denotes the vector concatenating all investment decisions K_g, $g \in \mathcal{G}$ and that these variables are upper bounded by a limit that we denote by L. We also denote by $\mathcal{G} \cup \{d\}$ the set of all market agents, including the consumer. Finally, we remind readers that every agent's ambiguity set, $\mathcal{B}_{\epsilon_a}(\hat{\mathbb{P}}_a)$, depends on the realization of market prices, which themselves depend implicitly on invested capacities $\boldsymbol{K}$. Therefore, in the remainder of this section, we make this dependence explicit by denoting ambiguity sets by $\mathcal{B}_{\epsilon_a}(\hat{\mathbb{P}}_a(\boldsymbol{K}))$.

Finally, $Int(A)$ will denote the interior of a given set A.

Assumption H1. *For any capacity mix $\boldsymbol{K} \in [0, L]^{|\mathcal{G}|}$, the interior of the intersection of all market agents' ambiguity sets is not empty:*

$$Int\left(\bigcap_{a\in\mathcal{G}\cup\{d\}} \mathcal{B}_{\epsilon_a}(\hat{\mathbb{P}}_a(\boldsymbol{K}))\right) \neq \varnothing. \tag{22}$$

We now present our existence result:

Proposition 1. *Under Assumption H1, Problem P has a solution.*

A proof is proposed in Appendix B.2.

3 Profits, welfare, and contracts under ambiguity

3.1 On the difference between ambiguity and risk aversion from a modeling perspective

We remind readers of the expression of the worst expectation of the second-stage cost that producer g incurs:

$$\sup_{\mathbb{Q}\in\mathcal{B}_{\varepsilon_g}(\hat{\mathbb{P}}_g)} \mathbb{E}_{\mathbb{Q}}\left[l_g\left(K_g, W_g, \xi_g(\boldsymbol{K}, \boldsymbol{W})\right)\right]. \tag{23}$$

At first sight, this expression is very close to that of a risk-averse agent valuing risk via a coherent risk measure (Föllmer and Schied [2002]),

$$\sup_{\mathbb{Q}\in\mathcal{M}_g} \mathbb{E}_{\mathbb{Q}}\left[l_g\left(K_g, W_g, \xi_g\right)\right], \tag{24}$$

where the so-called risk set $\mathcal{M}_g$ is a compact and convex set of probability measures. Given that set $\mathcal{B}_{\varepsilon_g}(\hat{\mathbb{P}}_g)$ is also compact and convex, one could consider our modeling of aversion to ambiguity as a particular instance of risk aversion when risk is measured with a coherent risk measure. This is not always true, mainly because risk set $\mathcal{M}_g$ is fixed and does not depend on second-stage parameters and profits nor on first-stage investment or contracting decisions. This feature makes it possible to invoke the envelope theorem to derive the classical complementarity conditions inherent to the investment decision under risk aversion, which states that the producer invests only if it trusts that it can recoup the investment cost via the risk-adjusted expectation of its second-stage margins (estimated with scarcity rents). This condition breaks with DRO because set $\mathcal{B}_{\varepsilon_g}(\hat{\mathbb{P}}_g)$ depends explicitly on second-stage uncertain prices. It also depends on the investment decisions via the formation of market prices according to the merit-order logic. We remind readers that these prices do intervene in the definition of the empirical probability measure $\hat{\mathbb{P}}_g$ and that this measure is built based on the realization of spot prices (in particular). Of course, this reasoning carries over to the consumer. Overall, these observations highlight the fundamental difference between risk and ambiguity aversion in our setting.

3.2 Ambiguity-adjusted profits and welfare in a competitive market

It is well known that under pure and perfect competition, market prices allow producers to recoup exactly their total supply cost, provided that marginal capital and operational costs are constant. In other words, producers' profits should equal zero in perfectly competitive markets. This property also holds when agents manage risk, even when they are risk-averse and value risk via coherent risk measures. In particular, it can be shown that, in the latter case, risk-adjusted profits are always null at equilibrium (see, for instance, Ralph and Smeers [2011], Abada et al. [2017a], Ferris and Philpott [2022] and related articles). The natural question is whether this result also applies when agents are ambiguity-averse. The main objective of this section is to show that the answer is yes, at least in our setting.

To do so, we first need to properly define what we mean by ambiguity-averse profits. As explained in Section 2.2.1, the objective function of producer $g \in \mathcal{G}$, who minimizes its capital cost plus its worst expectation of the operational cost, can be reformulated via Lemma 1 into Problem (9). Therefore, it is natural to define producer g's ambiguity-adjusted profit as:

Definition 2. *Producer g's ambiguity-adjusted profit is defined as*

$$AAP_g = -\Big(C_g K_g + \lambda_g \varepsilon_g + \frac{1}{N}\sum_i s_{ig}\Big). \tag{25}$$

A similar derivation of the consumer surplus and social welfare at equilibrium follows. We define the concept of an ambiguity-adjusted consumer surplus following similar logic:

Definition 3. *Ambiguity-adjusted consumer surplus is defined as:*

$$AACS = -\Big(\lambda_d \varepsilon_d + \frac{1}{N}\sum_i s_{id}\Big). \tag{26}$$

We can now define ambiguity-adjusted social welfare as the sum of the (ambiguity-adjusted) producer's profit and consumer surplus:

Definition 4. *Ambiguity-adjusted social welfare is then defined as:*

$$AASW = \sum_{g \in \mathcal{G}} AAP_g + AACS. \tag{27}$$

We now provide a result indicating that profits under ambiguity are always null at equilibrium.

Proposition 2 (Nullity of generator's profits)**.** *For any solution to equilibrium Problem P, ambiguity-adjusted profits are null:*

$$AAP_g = 0, \quad \forall g \in \mathcal{G}. \tag{28}$$

The proof is provided in Appendix B.3. At equilibrium, the ambiguity-adjusted industry profit is null. This is indeed an adaptation of the standard condition for competitive markets in the context of ambiguity and imperfect information. We draw the reader's attention to the fact that this condition holds even with the existence of financial contracts, reflecting a form of the absence of an arbitrage constraint in the financial market under ambiguity, as discussed in Section 2.2.1.

3.3 Contracts and ambiguity

It is widely admitted that, when properly designed, contracts can mitigate the detrimental impact of risk on welfare and investments, *when agents are averse to risk* (de Maere d Aertrycke and Smeers [2013], Ehrenmann and Smeers [2011], Abada et al. [2017b] and similar studies). This capacity holds not only for futures of forward contracts but also for some spot-indexed contracts, such as baseload or peakload contracts (Abada and Ehrenmann [2023], de Maere d Aertrycke et al. [2017]). In this section, we discuss the impact of hedging contracts when agents are averse to ambiguity.

3.3.1 Some contracts might heighten ambiguity

In a risk-averse setting, contracts can mitigate risk when they are correlated negatively with an agent's revenue. In our setting, where the revenue derived from a contract p^2_{ic} is stochastic (because the contract could be indexed on the spot-market price, for instance), agents will naturally include these contracts' revenues in the set of ambiguous parameters, as in (6) and (16). Therefore, as such, contracts might add to the ambiguity. We can illustrate this effect mathematically by considering producer g's ambiguity-adjusted profit,

$$AAP_g = -\Big(C_g K_g + \lambda_g \varepsilon_g + \frac{1}{N}\sum_i s_{ig}\Big), \tag{29}$$

and complementarity relation (10b) that explains the formation of variables s_{ig},

$$\begin{aligned} \forall i \in \mathcal{N}, \qquad s_{ig} \geq & \sum_t (H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt})(c_{igt} - p_{it}) + \sum_t \gamma^1_{igt}\bar{A}_g - \gamma^2_{igt}\underline{A}_g \\ & + \sum_c (W_{gc} - \gamma^1_{igc} + \gamma^2_{igc})(p^1_c - p^2_{ic}) + \sum_c \gamma^1_{igc}\bar{B}_g - \gamma^2_{igc}\underline{B}_g, \end{aligned}$$

which we can reformulate as: $\forall i \in \mathcal{N}$,

$$\begin{aligned} s_{ig} \geq & \sum_t H_t x'_{igt}(c_{igt} - p_{it}) + \sum_t \gamma^1_{igt}\Big[\bar{A}_g - (c_{igt} - p_{it})\Big] + \gamma^2_{igt}\Big[(c_{igt} - p_{it}) - \underline{A}_g\Big] \\ & + \sum_c W_{gc}(p^1_c - p^2_{ic}) + \sum_c \gamma^1_{igc}\Big[\bar{B}_g - (p^1_c - p^2_{ic})\Big] + \sum_c \gamma^2_{igc}\Big[(p^1_c - p^2_{ic}) - \underline{B}_g\Big]. \end{aligned} \tag{30}$$

Because γ^1_{igt} and γ^2_{igt} are non-negative and because

$$\begin{pmatrix} \underline{A}_g \\ \underline{B}_g \end{pmatrix} \leq \xi_{ig} = \begin{pmatrix} c_{igt} - p_{it} \\ p^1_c - p^2_{ic} \end{pmatrix} \leq \begin{pmatrix} \bar{A}_g \\ \bar{B}_g \end{pmatrix}, \tag{31}$$

we conclude that the presence of contracts increases s_{ig}, which in turn reduces the ambiguity-adjusted profit as formulated in (29). The natural consequence is that such ambiguous contracts will be avoided by market agents. Of course, this result is driven by the fact that the presence of contracts does not change the support of random spot prices ($\underline{A}_g$ and $\overline{A}_g$). Therefore, if one wants contracts to contribute to mitigating ambiguity, one must propose mechanisms that can reduce the support of ambiguity. This is the task to which we now turn.

3.3.2 Reducing ambiguity with Contracts for Difference

In this section we adapt our model to accommodate a type of contract that mitigates the effect of ambiguity and will likely be signed by ambiguity-averse market agents, namely Contracts for Difference (CfDs hereafter). CfDs are interesting in our context as they erase the ambiguity perceived in market data. In its simple design, a CfD is a contractual exchange between producers and consumers that swaps the market price with a fixed price named the "strike" price.[2] Such arrangements are usually signed for large capital-intensive investments such as nuclear reactors.[3] In our proposal, the CfD is contracted between a generator g and the representative consumer based on a strike price S and a covered capacity K^S_g.[4] In a nutshell, we consider a basic CfD agreement: a producer g endowed with a CfD receives the strike price S irrespective of the market price. The difference between the spot-market price and the strike must be settled between the producer and the consumer: if the spot price exceeds S in realization i and period t, the producer pays the difference $p_{it} - S$, multiplied by the production, x_{igt}, to the consumer. If, on the other hand, the spot price falls below S, the consumer pays $(S - p_{it}).x_{igt}$ to the producer.

To keep the length of the paper reasonable, we relegate the formulation of this problem to Appendix C, as it reveals some redundancies with our base model of Section 2. We simply mention here that, by erasing the price risk, the CfD mitigates the extent of ambiguity as it reduces the size of the support of ambiguous parameters related to the spot market price. Therefore, we expect this instrument to be more efficient at fostering investments and increasing welfare than

[2] Actually, such CfDs erase all uncertainty, and hence both ambiguity and risk. A relaxed version where ambiguity is reduced but risk remains would be to replace the strike price with a corridor $[X, Y]$: when the spot price p lies between X and Y, the producer receives the price. When $p \leq X$, it receives X and when $p \geq Y$, it receives Y.

[3] For example, in 2016 EDF, the French nuclear producer, signed a CfD with the UK government for the development of the 3.2 GW Hinkley Point C reactor with a strike price of 92 £/MWh (2012 prices).

[4] Another version would be to cover a volume of energy; this is equivalent in our framework. The limitation with capacity covered by the CfD is needed to ensure the boundedness of our variables, especially if the strike is strictly above the total cost. For ease of exposition, we assume that the generator under the CfD does not invest in uncovered physical capacity.

other contracts whose payoffs are stochastic. Of course, the effect would be similar with any other kind of contract that removes ambiguity, such as futures or forward contracts.

4 A numerical application to the French power system

In this section we illustrate the effects of ambiguity on welfare and capacity investment in a stylized representation of the French power system. We highlight how ambiguity aversion, if sufficiently high, destroys all incentives to invest and how spot-related contracts are unable to correct this phenomenon when they are ambiguous—but a CfD or a futures contract might.

4.1 Data and calibration

We focus on the main power-producing technologies in France: nuclear reactors (denoted by Nuclear), gas-fired load-following power plants (Mid gas), open- and closed-cycle gas turbines (OCGT and CCGT), and fuel-oil-powered plants for peak production (Fuel Oil). We deliberately ignore coal-fired plants as they are being progressively closed in Europe. Solar and onshore/offshore wind production are assumed to be exogenous. Therefore, we subtract their production from the load and focus instead on residual demand. We build $N = 60$ realizations (or scenarios) for residual demand D_{it} and the variable cost of each generator c_{igt}. This number is selected to keep the simulation time for our models reasonable. The operational phase of the second stage is composed of three timeblocks $t \in \{1, 2, 3\}$, with respective durations accounting for 20%, 70%, and 10% of the 8760 hours for the year (i.e. $H_1 = 1752$, $H_2 = 6132$, and $H_3 = 876$). Therefore, our modeling of time represents one typical year of operations. We now explain how to populate the 60 scenarios.

- Regarding the residual demand D_{it}, we use a dataset of past hourly demand and wind and PV (solar photo-voltaic) production in France between 2015 and 2024 (10 years of data), which we obtain from the transparency platform of the European Network of Transmission System Operators ENTSO-E.[5] For each year of data, we induce some randomness in the residual load by shifting the solar and wind production randomly by one or two weeks. Then, for each of these new vectors of hourly data, we use the first 20% quantile (of residual demand) to generate data for timeblock t_1. Similarly, data from between the 20% and 90% quantiles will be used for timeblock t_2 and data from the last 10% quantile will be used for timeblock t_3. This procedure preserves time correlations and allows us to populate the 60 scenarios required. Figure 2 depicts a histogram of the obtained residual demand in each period.

- Regarding variable costs c_{igt}, we make use of the following expression,

$$\forall i \in \mathcal{N}, g \in \mathcal{G}, t \in \mathcal{T}, \qquad c_{igt} = c_{bc,gt} + \delta_{ig} + e_g . p_i^{CO_2},$$

[5] See https://transparency.entsoe.eu/.

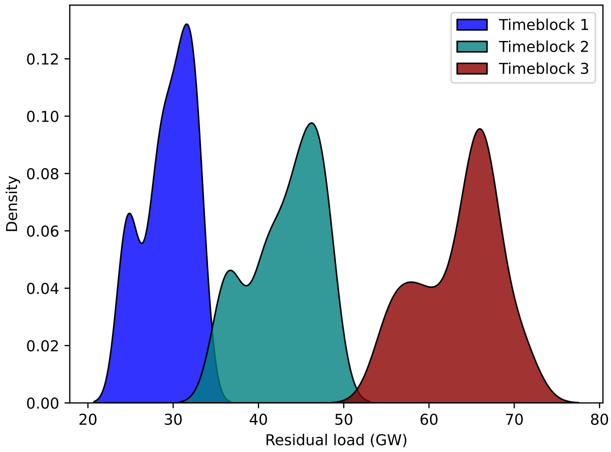

Figure 2: Histogram of the historical residual demand in the three timeblocks.

where $c_{bc,gt}$ is a technology-dependent base case operational cost (or OPEX, comprising the fuel cost and Operations and Maintenance cost), which we perturb by adding a random noise δ_{ig} calibrated to reflect the variability of input commodity prices as reported in CRE [2024], e_g is the carbon content of the technology, and $p_i^{CO_2}$ a random carbon price.[6]

- Regarding spot-indexed contracts, we model two instruments: i) a *baseload* contract which hedges the second-stage electricity price in timeblock 1 (low residual demand), i.e, $p^2_{i,\text{baseload}} = p_{i1}$, and ii) an *average* contract which hedges the mean power price over the whole year in each realization $p^2_{i,\text{average}} = \sum_t H_t p_{it}/8760$. We consider such instruments because, on the one hand, they are in widespread use in the power economy today while, on the other hand, they have been shown in the literature on risk-averse agents to enhance welfare and foster investments.

As envisaged in the French regulatory regime today, CfDs are offered to hedge investments in nuclear or renewables plants. For ease of exposition of our results, we focus in the present paper on those pertaining to nuclear production. Therefore, we study a nuclear CfD covering capacity K^S_{Nuclear} = 48 GW, which we set at the installed capacity of nuclear production calculated when agents are ambiguity-neutral and perfectly competitive. We vary the strike price of the CfD by testing three values: 30, 70, and 110 €/MWh. We note, in passing, that the latter value is the one that the French nuclear incumbent (EDF) secured prior to investing in the Hinkley Point C reactor in the UK (accounting for inflation and exchange rates).

Capital costs are transformed into annuities using expenditures and the lifetime of generation assets taken from Pietzcker et al. [2021], at a 4% interest rate. Table 1 summarizes our data for producers.

[6] The random carbon price $p_i^{CO_2}$ is uniformly drawn from $\{60, 80, 100\}$€/tCO_2, lying in the range of observed prices of the EU-ETS since 2022 (CRE [2024], p. 34).

	CAPEX (€/MW/year)	**Base case variable cost** (€/MWh)	**Carbon content** (g/kWh)
Nuclear	132605	30	0
CCGT	45471	60	200
Mid gas	35366	100	300
OCGT	20209	210	350
Fuel oil	75785	150	400

Table 1: Cost data for production technologies.

The value of Lost Load PC is taken at 3000 €/MWh. The representative consumer is assumed to be risk- and ambiguity-neutral ($\varepsilon_d = 0$) because, on the one hand, it does not face physical investment decisions in our framework while, on the other hand, it is represented by the French State in the contracting phase, which has access to many more hedging securities than the rest of the power economy. On the production side, we assume no existing capacity prior to the investment stage (in other words, we adopt a green-field approach). We also assume that generators have the same level of aversion to ambiguity, which we denote by ϵ and vary in our simulations, but this does not imply that they face the same risk or ambiguity exposure because their profits vary. The support bounds for the ambiguous parameters $\tilde{\xi}_g$ and $\tilde{\xi}_d$ are calibrated at their element-wise extrema across realizations, accounting for a price cap on the spot price taken at the Value of Lost Load PC. Therefore, we have: $\bar{A}_g = \max_{i,t} c_{igt}$, $\underline{A}_g = \min_{i,t} c_{igt} - PC$, $\bar{B}_g = \bar{B}_d = PC$, $\underline{B}_g = \underline{B}_d = -PC$, $\bar{A}_d = PC$, and $\underline{A}_d = 0$.

We compare all our results with the case where agents are *averse to risk* but not to ambiguity. As alluded to above, this situation is now routinely modeled in the OR literature even in the case of market incompleteness. In this setting, we model risk aversion by the Conditional Value at Risk (CVaR), which is a particular instance of a coherent risk measure (Artzner et al. [1999]) and can be computed via linear programming (Rockafellar and Uryasev [2000]). We refer the reader to Appendix D, where we present the full resulting equilibrium model. Here again, producers exhibit the same level of risk aversion denoted by ϵ^{CVaR}, which we vary between 0 and 1, without necessarily facing the same risk exposure, and consumers are risk-neutral.

In total, we consider the following eight simulation cases (or benchmarks):

- Ambiguity-averse generators without financial contracts (denoted by "Ambi. - None" in our results).
- Ambiguity-averse generators with the 'baseload' contract ("Ambi. - Base").
- Ambiguity-averse generators with the 'average' contract ("Ambi. - Avg.").
- Ambiguity-averse generators with a CfD of strike x €/MWh for nuclear generation ("Ambi. - CfD x"). We remind readers that we undertake a sensitivity analysis with respect to the price of the CfD as in $x \in \{30, 70, 110\}$.

- Symmetrically, we also consider the same cases for risk-averse generators and denote them, respectively, as "Risk - None", "Risk - Base", "Risk - Avg.", and "Risk - CfD x".

All complementarity problems are solved using the commercial PATH solver in their extensive form.

4.2 Results

This section presents our main findings. We focus on the comparative effects of ambiguity and risk on welfare, total capacity and capacity mix, and those of the considered financial contracts. Figure 3 presents our results regarding welfare. In the ambiguity-averse case (left and center panels), we find that welfare diminishes with the level of ambiguity aversion. Ultimately, the power economy collapses for extreme values of ambiguity aversion, with its welfare vanishing, except for cases with a CfD. Surprisingly, we observe that spot-indexed contracts do not run counter to this detrimental adverse effect of ambiguity. This is a consequence of their adding to ambiguity as they deliver uncertain revenue, as discussed in Section 3.3.1. Similarly, as can be seen in the center panel, a CfD with a strike price that is too low does not prevent welfare from vanishing (for instance, in our numerical example, 30 €/MWh is lower than the average variable cost for nuclear production and does not compensate for capital expenditures). On the other hand, CfDs with sufficiently high strike prices mitigate the diminishing effects of high ambiguity aversion on welfare.

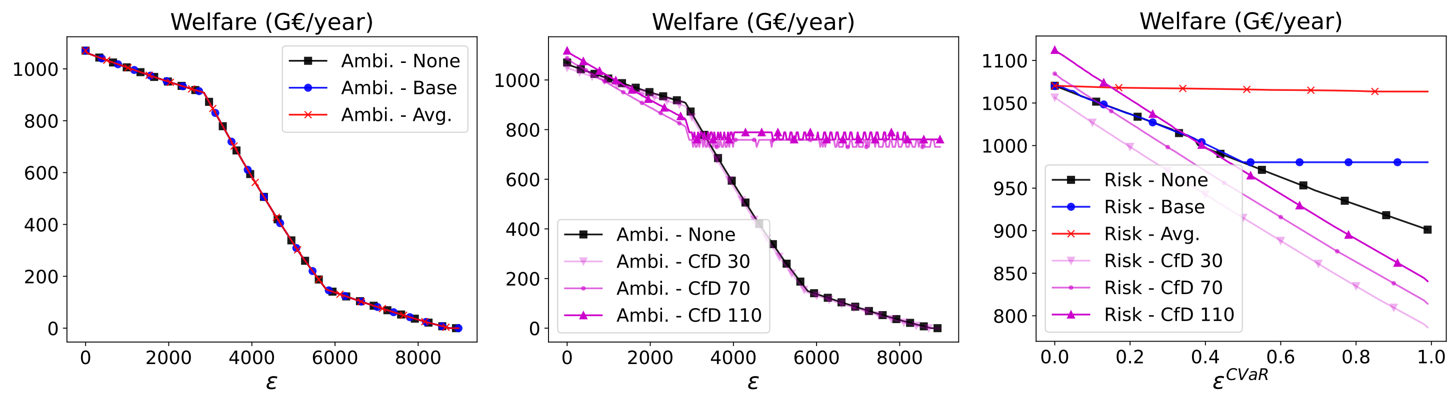

Figure 3: Welfare with ambiguity-averse agents with spot-indexed prices (Left), with CfDs (Center), and with risk-averse agents with and without CfDs (Right).

Our results obtained for the risk-averse cases (right panel of Figure 3) are in line with the literature. First, as risk aversion rises, welfare diminishes. This reduction is, however, much less detrimental to the economy than the effects of ambiguity aversion, as we observe that welfare does not reach zero for an extreme level of risk aversion. In other words, risk aversion does not make the power economy collapse but simply makes it less efficient. This is because an overly risk-averse investor focuses on the worst of historical realizations of its profit when valuing its project, whereas an overly ambiguity-averse investor would consider a hypothetical realization lying on an edge of the support for ambiguity, leading to much lower profits and almost no incentive to invest. Second, spot-indexed contracts are, contrary to what we observed for ambiguity, welfare-augmenting, as the "average" contract provides nearly perfect risk transfers such that welfare remains almost constant in our example. Third, CfDs, if calibrated properly, also increase welfare.

Results related to the invested physical capacity are reported in Figure 4. They confirm the insights provided by our analysis of welfare. Aversion to ambiguity harms investments but CfDs with sufficient strike prices can mitigate this effect. In line with the literature, we find that spot-indexed contracts limit the issue when there is aversion to risk, whereas they do not do so under ambiguity. In Appendix A (Figure 5), we provide additional insights regarding the mix of invested capacities under ambiguity. These insights highlight that the impact of ambiguity on the system's capacity translates to all technologies. We also observe that a nuclear CfD with a high strike price does preserve nuclear capacity at the expense of some other technologies (OCGT and CCGT) because greater nuclear capacity reduces inframarginal rents at peak production. Naturally, the effect is the opposite with a low strike price.

Why are spot-indexed contracts inefficient at mitigating the detrimental impact of ambiguity? In reality, in a risk-averse context, these contracts allow market agents to hedge their profits according to some realization-dependent outcome because returns in the spot and financial markets can be made correlated via the same realization of some uncertain parameter, in our case the spot-market price. This is why such contracts will be sought by (risk-averse) investors. With ambiguity-averse agents, this correlation is lost: returns in the spot and financial markets become decoupled as the worst-case distribution of the uncertain returns may not be the same in both markets. Putting it differently, the worst-case distribution of spot returns may be calculated with a support of the spot-market price that does not change with the introduction of spot-indexed contracts. Therefore, these contracts may fail to mitigate the impact of ambiguity aversion. Only suppression of the price risk, such as that provided by a CfD, can do so. Naturally, in our framework, this result stems from the fact that the presence of spot-indexed contracts does not alter the support of the distribution of the random spot price ($\underline{A}_g$ and $\overline{A}_g$). We discuss the implications of this mechanism in Section 5.

5 Discussion

During the energy transition, the detrimental effect of risk on capacity investment in power systems and its management through long-term contracts have become a central focus. The recent energy crisis in Europe, the surge in renewables capacities going hand-in-hand with zero

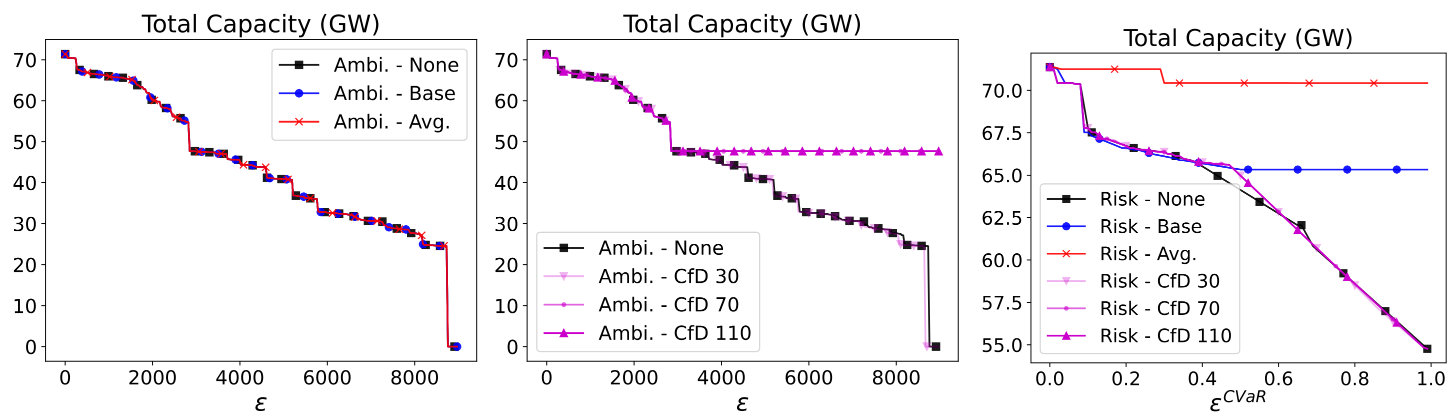

Figure 4: Total physical capacity with ambiguity-averse agents with spot-indexed contracts (Left), with CfDs (Center), and with risk-averse agents with and without CfDs (Right).

or negative spot prices, or the impact of market design reforms cast doubt on the informative power of historical market data or the empirical distribution of uncertain parameters such as prices for future revenues. The result is increased aversion to ambiguity. Yet, decarbonization policies rely on both electrification and cutting emissions from the power sector, and new green investments are required. Therefore, understanding how ambiguity affects investments, how it can be mitigated, and how it compares with classical aversion to risk is of paramount importance.

We propose a novel model of the power economy with price-taking agents who are ambiguity-averse and may exchange contracts with each other. The model relies on the traditional two-stage capacity-expansion framework in equilibrium, to which we add a representation of ambiguity in the decision process. Aversion to ambiguity is modeled via Wasserstein distributionally robust optimization, inspired by the seminal work of Mohajerin Esfahani and Kuhn [2018]. We recast this infinite dimensional problem into linear programs with little need to strengthen traditional assumptions for capacity-expansion models. We provide an existence result and successfully apply our model to a simple representation of the French power system, which is characterized by a variety of production technologies.

We prove a form of the welfare theorem by assessing the nullity of the ambiguity-adjusted producers' profits and discuss the adaptation of the absence of an arbitrage condition in the financial trade to accommodate ambiguity. Through our analysis of the complementarity conditions, we are able to indicate why some contracts with uncertain remuneration fail to reduce ambiguity. Practically, this implies that such contracts might be unable to mitigate the detrimental impact of ambiguity on welfare and investment as they would classically do in a risk-averse environment. Results would have been more positive if some spot-indexed contracts were able to reduce the support of the spot-price distribution. Finding such a mechanism would make it possible to reap the benefits of contracts even in the presence of aversion to ambiguity. We leave the investigation of such schemes to future research. On the other hand, we indicate how some existing schemes, such as CfDs, effectively restore efficiency when there is ambiguity, primarily because these contracts erase the price risk and thus the ambiguity. Their main drawback, however, is that they might distort competition. Indeed, producers benefiting from a profitable CfD might no longer react swiftly to the price signal, harming the efficiency of spot markets. In fact, similar contractual arrangements for solar and wind production—such as Power Purchase Agreements (PPAs) or Feed-In Tariffs—are believed to be a key driver of the crisis of negative prices in Europe as some contracted renewable production is offered irrespective of the spot market price. This could induce massive oversupply in hours of low demand, especially when flexibility and storage capacity are insufficient. Such an effect can, in turn, increase the level of ambiguity aversion of market participants. Therefore CfDs with corridors might induce an acceptable balance between reducing ambiguity and keeping producers reactive to price signals.

We see our work as a very first step towards integrating ambiguity with respect to the distribution of some uncertain parameters in models of capacity expansion. Therefore, our models are intentionally simplified, omitting many technical aspects of the power economy. Yet, we believe that these models serve as solid proof of concept, demonstrating the feasibility of this endeavor. With our models, we show that some instruments may succeed in addressing risk but fail with respect to ambiguity. This raises the more fundamental question of how risk-born and ambiguity-born incentives among agents can be disentangled, which should require additional research. Future research could also investigate the possibility of adapting the ambiguity sets of agents that depend on the realizations of data (the so-called adaptative distributionally robust framework), the multiplicity of equilibria under ambiguity, or the generalizability of our findings with other kinds of contracts.

A Numerical result - Invested capacity per generator

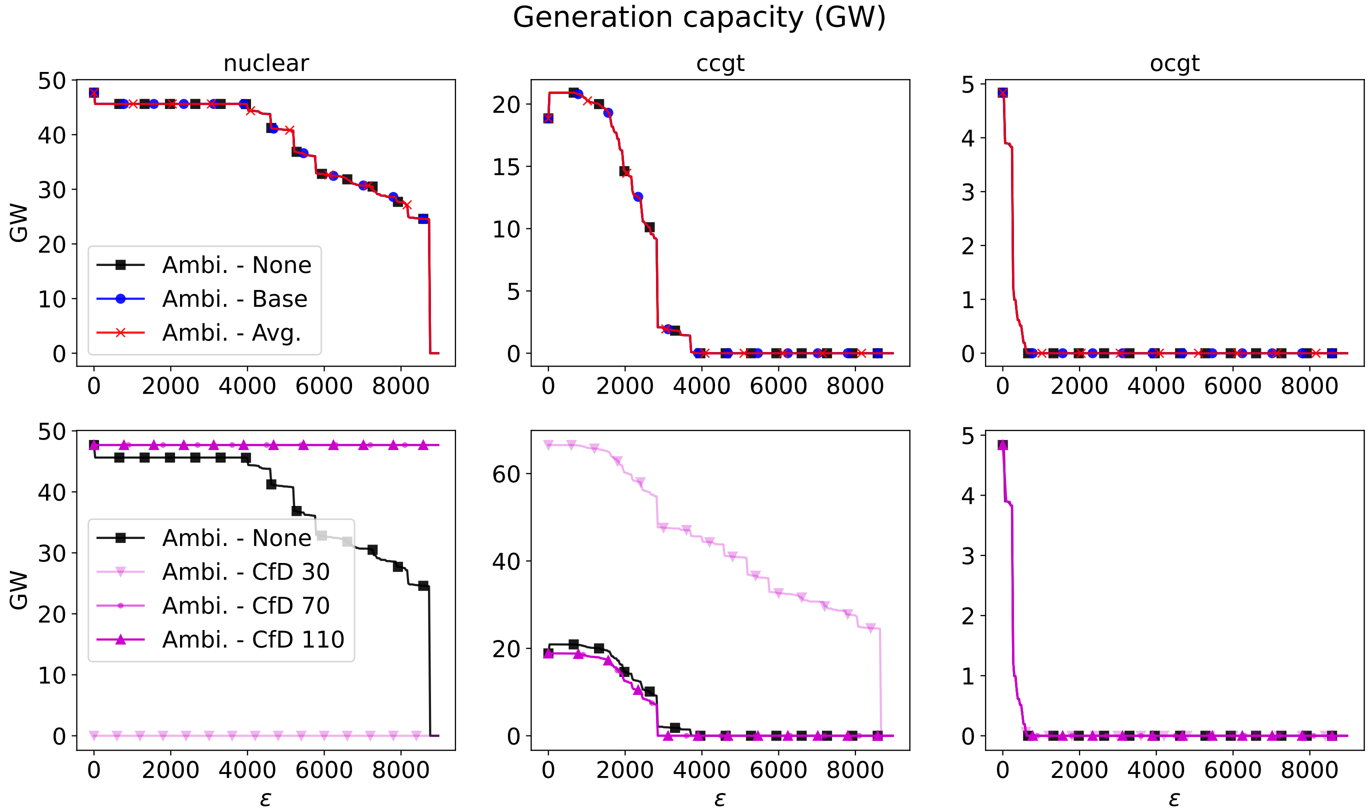

Figure 5: Invested physical capacity per generation technology with ambiguity-averse agents without (Up) and with (Bottom) a CfD on nuclear capacity. Fuel-oil-powered plants are not invested in

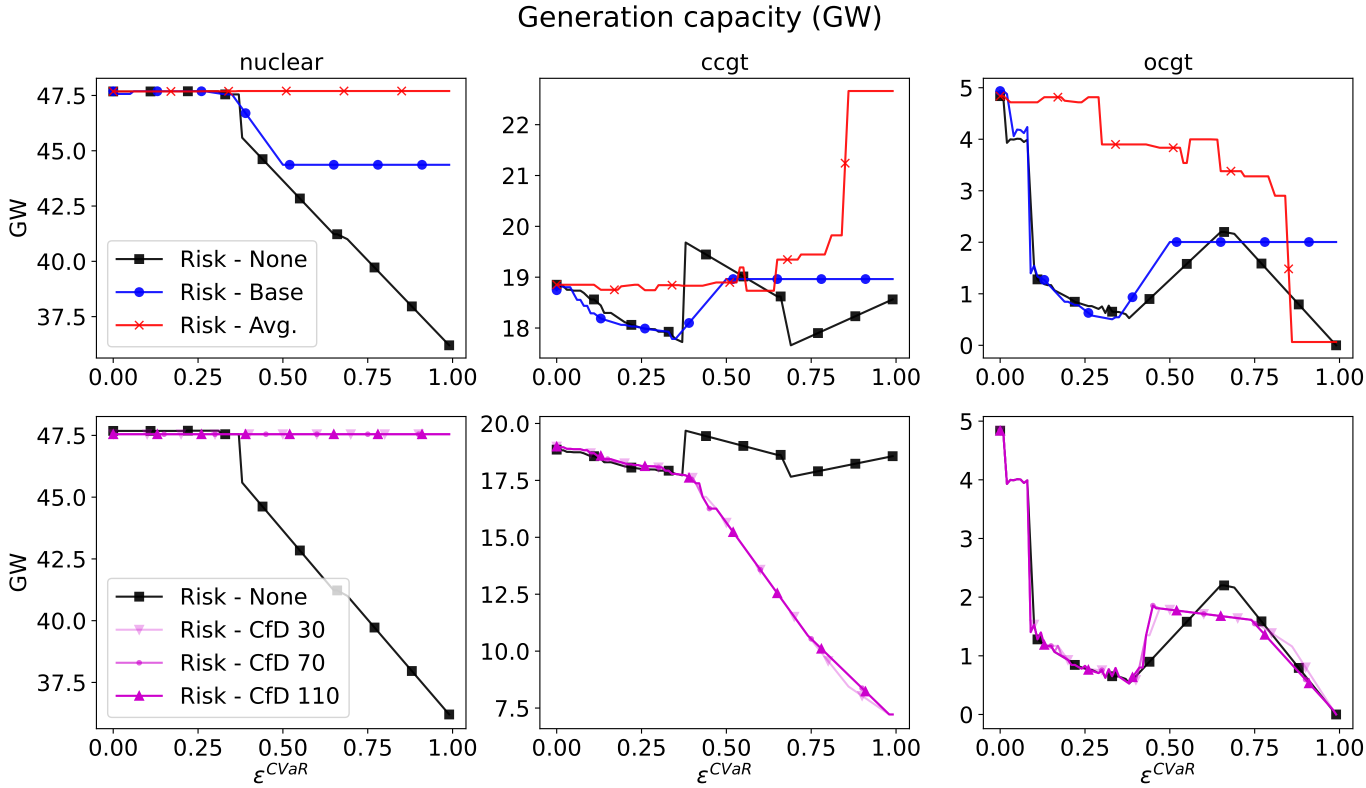

Figure 6: Invested physical capacity per generation technology with risk-averse agents without (Up) and with (Bottom) a CfD on nuclear capacity. Fuel-oil-powered plants are not invested in.

B Proofs

B.1 Proof of Lemma 1

Proof. The support set Ξ is convex and closed. l is an infimum of linear functions of parameter ξ such that l is concave and is not $-\infty$ on Ξ because the feasible set of the second stage is assumed to be non-empty. Therefore, Theorem 4.2 in Mohajerin Esfahani and Kuhn [2018] applies and Problem (2) reduces to

$$\begin{aligned}
&\inf_{\lambda, s_i, z_i, \nu_i \in \mathbb{R}} \lambda \varepsilon + \frac{1}{N}\sum_{i=1}^{N} s_i \qquad (32)\\
&\text{s.t. } \forall i \in \mathcal{N}, \qquad [-l]^*(z_i - \nu_i) + \sigma_\Xi(\nu_i) - \langle z_i, \hat{\xi}_i \rangle \leq s_i\\
&\forall i \in \mathcal{N}, \qquad ||z_i||_* \leq \lambda,
\end{aligned}$$

where $||.||_*$ is the dual norm of the underlying norm of the Wasserstein metric, which, we remind the reader, is here the $\mathcal{L}_1$ norm. Therefore, $||.||_*$ is the infinite norm $||.||_\infty$. Function σ_Ξ is the support function for Ξ and $[-l]^*$ is the conjugate function for $-l$. By definition, for all $i \in \mathcal{N}$,

$$\begin{aligned}\sigma_\Xi(\nu_i) &= \sup_{\underline{D} \le \xi \le \bar{D}} \langle \nu_i, \xi \rangle \\ &= \inf_{\substack{\gamma_i^1, \gamma_i^2 \ge 0 \\ \nu_i = \gamma_i^1 - \gamma_i^2}} \langle \gamma_i^1, \bar{D} \rangle - \langle \gamma_i^2, \underline{D} \rangle,\end{aligned}$$

where γ_i^1, γ_i^2 are the dual variables associated with $\xi \le \bar{D}$ and $\xi \ge \underline{D}$, respectively, and the second line reflects the strong duality in this linear program. Moreover, also by definition of the conjugate function, for all $i \in \mathcal{N}$,

$$\begin{aligned}[-l]^*(z_i - \nu_i) &= \sup_{\xi \in \mathbb{R}^m} \left(\langle z_i - \nu_i, \xi \rangle + \inf_{W y_i \ge h} \langle Q^T y_i + \alpha, \xi \rangle \right) \\ &= \inf_{W y_i \ge h} \sup_{\xi \in \mathbb{R}^m} \langle Q^T y_i + \alpha + z_i - \nu_i, \xi \rangle \\ &= \begin{cases} 0 & \text{if } W y_i \ge h \text{ and } Q^T y_i + \alpha = -z_i + \nu_i \\ -\infty & \text{otherwise} \end{cases},\end{aligned}$$

where the second line follows from the minimax theorem, as the feasibility set of the second stage is compact and convex. Incorporating the previous results into Problem (32), replacing z_i and ν_i with $-Q^T y_i - \alpha + \gamma_i^1 - \gamma_i^2$ and $\gamma_i^1 - \gamma_i^2$, respectively, and rewriting $||z_i||_\infty \le \lambda$ as $-\lambda \le z_{ik} \le \lambda$ for all $k \in \{1, ..., m\}$ ends the proof. □

B.2 Proof of Proposition 1

Proof. As explained in Section 2.2.3, it remains to demonstrate that contract volumes W_{gc} and W_{dc} are bounded for equilibrium to exist. We adapt the proof of the existence of equilibria with convex risk measures elaborated in de Maere d'Aertrycke and Smeers [2013] to our ambiguity framework. For clarity and readability, we intentionally retain some of the notations and wording used in that article.

In this proof, we amend our index notation slightly by making index g denote any market agent—a producer or the consumer—$g \in \mathcal{G} \cup \{d\}$. We begin the proof by noting that the problem for any agent $g \in \mathcal{G} \cup \{d\}$ takes the form:

$$\inf_{\substack{W_g \in \mathbb{R}^{|\mathcal{C}|} \\ K_g \ge 0}} \sup_{\mathbb{Q} \in \mathcal{B}_{\epsilon_g}(\hat{\mathbb{P}}_g(\boldsymbol{K}))} \mathbb{E}_\mathbb{Q}[-\pi_g] + C_g K_g + \sum_c W_{gc} \left(p_c^1 - \mathbb{E}_\mathbb{Q}[p_c^2] \right), \tag{33}$$

where π_g denotes agents g's second-stage profits accrued from the physical spot market and where we make $C_d = 0$. For any capacity mix $K \in [0, L]^{|\mathcal{G}|}$, we introduce the set of not overly attractive contract prices for agent $g \in \mathcal{G} \cup \{d\}$ as:

$$P_g(\boldsymbol{K}) = \left\{ p^1 \in \mathbb{R}^{|\mathcal{C}|}; \exists \mathbb{Q} \in \mathcal{B}_{\epsilon_g}(\hat{\mathbb{P}}_g(K)), \forall c \in \mathcal{C}, p^1_c = \mathbb{E}_{\mathbb{Q}}[p^2_c] \right\}. \tag{34}$$

$P_g(\boldsymbol{K})$ is a compact and convex set because $\mathcal{B}_{\epsilon_g}(\hat{\mathbb{P}}_g(\boldsymbol{K}))$ is convex and $\mathbb{E}_{\mathbb{Q}}[p^2_c]$ is bounded because p^2_c is indexed on the spot price, which is itself bounded. We denote by $\bar{P}(\boldsymbol{K})$ the intersection of the sets $P_g(\boldsymbol{K})$ for all agents. Assumption H1 implies that the interior of $\bar{P}(\boldsymbol{K})$ is not empty.

The sets of not overly attractive financial prices allow us to reformulate the worst-case second-stage outcome at a given capacity mix, $\sup_{\mathbb{Q} \in \mathcal{B}_{\epsilon_g}(\hat{\mathbb{P}}_g(\boldsymbol{K}))} \mathbb{E}_{\mathbb{Q}}[-\pi_g] + C_g K_g + \sum_c W_{gc} \left(p^1_c - \mathbb{E}_{\mathbb{Q}}[p^2_c]\right)$, as (some dual variables are written next to their constraints)

$$\begin{aligned}
= \sup_{\mathbb{Q} \in \mathcal{B}_{\epsilon_g}(\hat{\mathbb{P}}_g(\boldsymbol{K}))} \quad & \mathbb{E}_{\mathbb{Q}}[-\pi_g] + C_g K_g & & \text{if } p^1 \in P_g(\boldsymbol{K}) \\
\text{s.t.} \quad & \forall c \in \mathcal{C}, p^1_c = \mathbb{E}_{Q}[p^2_c] & [W_{gc}] & \\
= +\infty & & & \text{otherwise.}
\end{aligned} \tag{35}$$

The second-stage profits π_g are bounded because they are accrued from the spot market. Therefore, the objective of the reformulated Problem (35) is finite whenever $p^1 \in P_g(\boldsymbol{K})$. Its feasibility constraint is convex and verifies strong Slater conditions when $p^1 \in Int(P_g(\boldsymbol{K}))$. Thus, the set of optimal dual variables in the reformulated Problem (35) is non-empty, convex, and compact. Inasmuch as financial positions W_{gc} act as dual multipliers of the reformulated problem's feasibility constraint, we have the following lemma:

Lemma 2. *For any capacity mix $\boldsymbol{K} \in [0, L]^{|\mathcal{G}|}$ and for any agent $g \in \mathcal{G} \cup \{d\}$, the set of optimal solutions to problem*

$$\inf_{W_g \in \mathbb{R}^{|\mathcal{C}|}} \sup_{\mathbb{Q} \in \mathcal{B}_{\epsilon_g}(\hat{\mathbb{P}}_g(\boldsymbol{K}))} \mathbb{E}_{\mathbb{Q}}[-\pi_g] + C_g K_g + \sum_c W_{gc} \left(p^1_c - \mathbb{E}_{\mathbb{Q}}[p^2_c]\right),$$

is non-empty, convex, and compact iff $p^1 \in Int(P_g(\boldsymbol{K}))$.

A competitive equilibrium of our power economy, if it exists, is such that the financial market clears, $\forall c \in \mathcal{C}, \sum_{g \in \mathcal{G} \cup \{d\}} W_{gc} = 0$, and the invested capacities and financial positions are optimal for all agents. It can easily be shown that the financial-clearing constraint, $\forall c \in \mathcal{C}, \sum_{g \in \mathcal{G} \cup \{d\}} W_{gc} = 0$, can be replaced by modeling a *financial-market agent* acting as an arbitrageur whose decision variables are the contract prices p^1_c and whose objective is to minimize contract volume shortfalls as follows:

$$p^1 \in \text{argmin}_{q \in \mathbb{R}^{|\mathcal{C}|}} \sum_c -q_c \left(\sum_{g \in \mathcal{G} \cup \{d\}} W_{gc} \right). \tag{36}$$

Therefore, a Nash equilibrium $(\boldsymbol{K}, \boldsymbol{W}, p^1)$ of the (new) game where agents $\mathcal{G} \cup \{d\}$ select physical capacity $\boldsymbol{K}$ and financial positions $\boldsymbol{W}$, which together solve Problem (33), and where the financial-market agent selects price contracts p^1_c which solve Problem (36), is also an equilibrium of our Problem P. Notably, if it exists, a Nash equilibrium of this new game ensures that the financial market clears.

Let us now consider a parameter $r > 0$ and the *truncated game* where financial positions are bounded by r as in $\forall (g, c) \in (\mathcal{G} \cup \{d\}) \times \mathcal{C}$, $|W_{gc}| \leq r$, and where contract prices belong to the non-empty, convex, and compact set $\Delta = \prod_{c \in \mathcal{C}} [\min_i \; p^2_{ic} \; - \; 1, \max_i p^2_{ic} \; + \; 1]$. We remind readers that physical capacities already belong to the convex and compact set $[0, L]^{|\mathcal{G}|}$. Because the programs of all agents are convex in the strategies of the other agents and continuous in their strategy and because the set of strategies of the truncated game is non-empty, convex, and compact, we can invoke Debreu's theorem (Debreu [1952]) to state that there exists an equilibrium $(\boldsymbol{K}^r, \boldsymbol{W}^r, p^{1,r})$ of the truncated game. This equilibrium also verifies the financial market-clearing. Otherwise, the financial market agent selects price $p^{1,r}_c = \min_i \; p^2_{ic} \; - \; 1$ (or $p^{1,r}_c = \max_i \; p^2_{ic} \; + \; 1$) for the contracts that are not cleared, such that players position these contracts at $-r$ (resp. $+r$). The objective of the financial market agent is thus $-r(|\mathcal{G}| + 1)(\min_i \; p^2_{ic} \; - \; 1)$ (resp. $r(|\mathcal{G}| + 1)(\max_i \; p^2_{ic} \; + \; 1)$), which is clearly not optimal and thus absurd.

We then let r run to $+\infty$ and consider a sequence of equilibria of the truncated games $(\boldsymbol{K}^r, \boldsymbol{W}^r, p^{1,r})_r$. We wish to find some game m such that $\boldsymbol{W}^m$ is in the interior of the admissible set of financial positions, i.e., $\forall (g, c) \in (\mathcal{G} \cup \{d\}) \times \mathcal{C}$, $|W^m_{gc}| \; < m$. Because the agents solve convex minimization programs, the optimal financial positions $\boldsymbol{W}^m$ in the m truncated game would also be optimal in the initial game. Moreover, we already have that $p^{1,m}$ is bounded independently of m, as it belongs to Δ. In turn, the equilibrium $(\boldsymbol{K}^m, \boldsymbol{W}^m, p^{1,m})$ would be an equilibrium of our initial Problem P.

We turn now to showing the existence of such m. By definition, if the sequence $(\boldsymbol{W}^r)_r$ is bounded, such m exists. Suppose the sequence is unbounded. There then exists a player g with an unbounded sequence of financial positions. The objective function of player g is, for all r, large enough such that $W^r_g \neq 0$,

$$\mathbb{E}_{\mathbb{Q}}[-\pi_g(\omega)] + C_g K^r_g + ||W^r_g|| \sum_c \frac{W^r_{gc}}{||W^r_g||} \left(p^{1,r}_c - \mathbb{E}_{\mathbb{Q}}[p^2_c(\omega)] \right).$$

The first two terms of the objective are bounded *independently of* r. Moreover, for a given r, if there existed a distribution $\mathbb{Q}^r$ such that

$$\sum_c \frac{W^r_{gc}}{||W^r_g||} \left(p_c^{1,r} - \mathbb{E}_{\mathbb{Q}^r}[p_c^2(\omega)] \right) > 0,$$

then $\mathbb{Q}^r$ would be the worst-case distribution and the agent could achieve an infinite objective, as the norm of W^r_g runs to infinity, which is absurd. Therefore, for a sufficiently large r, we have

$$\forall \mathbb{Q} \in \mathcal{B}_{\epsilon_g}(\hat{\mathbb{P}}_g(\boldsymbol{K^r})), \sum_c W^r_{gc} \left(p_c^{1,r} - \mathbb{E}_{\mathbb{Q}}[p_c^2(\omega)] \right) \leq 0.$$

Now recall that the financial markets clear at a Nash equilibrium of the truncated game. Thus, for a sufficiently large r, there exists a (smallest) subset J_r of players with non-null positions which balance that of player g, i.e. $\sum_{j \in J_r} W^r_j = -W^r_g$. Following similar reasoning for positions W^r_j, we must have

$$\forall j \in J_r,\ \forall \mathbb{Q} \in \mathcal{B}_{\epsilon_j}(\hat{\mathbb{P}}_j(\boldsymbol{K^r})), \sum_c W^r_{jc} \left(p_c^{1,r} - \mathbb{E}_{\mathbb{Q}}[p_c^2(\omega)] \right) \leq 0. \tag{37}$$

Concatenating and then summing relations (37) for $j \in J_r$, we obtain

$$\forall \mathbb{Q} \in \bigcap_{j \in J_r} \mathcal{B}_{\epsilon_j}(\hat{\mathbb{P}}_j(\boldsymbol{K^r})),\ 0 \geq \sum_c \left(\sum_{j \in J_r} W^r_{jc} \right) \left(p_c^{1,r} - \mathbb{E}_{\mathbb{Q}}[p_c^2(\omega)] \right) = \sum_c -W^r_{gc} \left(p_c^{1,r} - \mathbb{E}_{\mathbb{Q}}[p_c^2(\omega)] \right),$$

where $\bigcap_{j \in J_r} \mathcal{B}_{\epsilon_j}(\hat{\mathbb{P}}_j(\boldsymbol{K^r}))$ is non-empty thanks to Assumption H1. Hence, as $(W^r_g)_r$ is unbounded, we can find some m for which $W^m_g \neq 0$ and

$$\begin{aligned}
&\forall \mathbb{Q} \in \mathcal{B}_{\epsilon_g}(\hat{\mathbb{P}}_g(\boldsymbol{K^m})), && \sum_c W^m_{gc} \left(p_c^{1,m} - \mathbb{E}_{\mathbb{Q}}[p_c^2(\omega)] \right) \leq 0 \\
&\forall \mathbb{Q} \in \bigcap_{j \in J_m} \mathcal{B}_{\epsilon_j}(\hat{\mathbb{P}}_j(\boldsymbol{K^m})), && \sum_c W^m_{gc} \left(p_c^{1,m} - \mathbb{E}_{\mathbb{Q}}[p_c^2(\omega)] \right) \geq 0.
\end{aligned}$$

Therefore, W^m_g defines a separating hyperplane between the convex and open set $Int\left(P_g(K^r)\right)$ and the convex set $\bigcap_{j \in J_r} Int(P_j(K^r))$. Thus, those sets are disjoints (see e.g., Boyd and Vandenberghe [2004] p. 50): $Int\left(P_g(K^m)\right) \cap \bigcap_{j \in J_m} Int(P_j(K^m)) = \varnothing$ and therefore $\bar{P}(K^r) = \varnothing$. This contradicts Assumption H1 and concludes the proof. □

B.3 Proof of Proposition 2

Proof. The ambiguity-adjusted profit generator $g \in \mathcal{G}$ earns can be written as $-(\lambda_g \varepsilon_g + 1/N \sum_i s_{ig} + C_g K_g)$, which we want to prove equals zero.

We focus on expressing $1/N \sum_i s_{ig}$.

Using (10k), $\alpha_{ig} = -1/N < 0$ such that, with (10b), for all $i \in \mathcal{N}$,

$$-\frac{1}{N}s_{ig} = \sum_t (H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt})\alpha_{ig}(c_{igt} - p_{it}) + \alpha_{ig}\gamma^1_{igt}\bar{A}_g - \alpha_{ig}\gamma^2_{igt}\underline{A}_g$$
$$+ \sum_c (W_{gc} - \gamma^1_{igc} + \gamma^2_{igc})\alpha_{ig}(p^1_c - p^2_{ic}) + \alpha_{ig}\gamma^1_{igc}\bar{B}_g - \alpha_{ig}\gamma^2_{igc}\underline{B}_g.$$

Focusing on the time-indexed variables, we have, for all $i \in \mathcal{N}$,

$$\begin{aligned}
&\sum_t (H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt})\alpha_{ig}(c_{igt} - p_{it}) + \sum_t \alpha_{ig}\gamma^1_{igt}\bar{A}_g - \alpha_{ig}\gamma^2_{igt}\underline{A}_g \\
&= \sum_t \mu'_{igt}x'_{igt} + H_t\beta^2_{igt}x'_{igt} - H_t\beta^1_{igt}x'_{igt} + \sum_t \alpha_{ig}(\gamma^2_{igt} - \gamma^1_{igt})(c_{igt} - p_{it}) + \sum_t \alpha_{ig}\gamma^1_{igt}\bar{A}_g - \alpha_{ig}\gamma^2_{igt}\underline{A}_g \\
&= \sum_t \mu'_{igt}x'_{igt} + H_t x'_{igt}\beta^2_{igt} - H_t x'_{igt}\beta^1_{igt} + \sum_t (\gamma^2_{igt} - \gamma^1_{igt})(\beta^2_{igt} - \beta^1_{igt}) \\
&= \sum_t \mu'_{igt}x'_{igt} - \lambda_g(\beta^1_{igt} + \beta^2_{igt}) \\
&= \sum_t \mu'_{igt}K_g - \lambda_g \sum_t (\beta^1_{igt} + \beta^2_{igt}),
\end{aligned}$$

where the first equality comes from (10g), the second comes from (10l) and (10m), the third comes from (10c) and (10d), and the last comes from (10a). Following steps similar to those used with the contract-indexed variables, we have

$$\begin{aligned}
&\sum_i \sum_c (W_{gc} - \gamma^1_{igc} + \gamma^2_{igc})\alpha_{ig}(p^1_c - p^2_{ic}) + \sum_{i,c} \alpha_{ig}\gamma^1_{igc}\bar{B}_g - \alpha_{ig}\gamma^2_{igc}\underline{B}_g \\
&= \sum_{i,c} \beta^2_{igc}[W_{gc} - \gamma^1_{igc} + \gamma^2_{igc}] + \sum_{i,c} \beta^1_{igc}[-W_{gc} + \gamma^1_{igc} - \gamma^2_{igc}] \\
&= \sum_{i,c} -\lambda_g(\beta^1_{igc} + \beta^2_{igc}),
\end{aligned}$$

where the first equality derives from (10i), (10n), and (10o) and the second derives from (10e) and (10f). Aggregating the results,

$$\begin{aligned}
\frac{1}{N}\sum_i s_{ig} &= -\sum_{i,t} \mu'_{igt}K_g + \lambda_g \left(\sum_{i,t} (\beta^1_{igt} + \beta^2_{igt}) + \sum_{i,c} (\beta^1_{igc} + \beta^2_{igc}) \right) \\
&= -C_g K_g - \lambda_g \epsilon_g,
\end{aligned}$$

where the first term of the second equality comes from the generator's rent covering its capital expenditures (10h) and the second term comes from (10j). This proves the nullity of the ambiguity-adjusted profit each generator earns. □

C The complementarity formulation of the economy with ambiguity-averse agents exchanging CfDs

In this section we amend the model we present in Section 2.2 to accommodate CfDs. All producers who do not benefit from the contract are modeled in the same way as in Section 2.2.1. For ease of exposition, we assume that only one producer g benefits from the CfD, but this assumption is not constraining in our framework as it can be easily relaxed. Consider, then, producer g, which is endowed with a CfD of strike S covering a maximum capacity K_g^S contracted with the representative consumer. In realization i, generator g still minimizes its cost, but its generation is now valued at the strike price, S. Its second-stage problem can hence be written as

$$l_g(K_g, W_g, \xi_{ig}) = \min_{x_{igt} \geq 0} \sum_t H_t(c_{igt} - S)x_{igt} + \sum_c (p_c^1 - p_{ic}^2)W_{gc} \tag{38}$$
$$\text{s.t. } \forall t \in \mathcal{T}, \qquad x_{igt} \leq K_g \qquad [\mu_{igt}].$$

In the first stage, producer g still minimizes its investment costs plus its worst-expected operational loss. It faces an ambiguous, uncertain parameter ξ_g which now accounts for a fixed spot price in the inframarginal rents (we still use notation ξ_g for uncertain parameters, with no risk of confusion):

$$\forall i \in \mathcal{N}, \xi_{ig} = \begin{pmatrix} c_{igt} - S \\ p_c^1 - p_{ic}^2 \end{pmatrix} \in \mathbb{R}^{T+|\mathcal{C}|}. \tag{39}$$

The bounds of the uncertainty set are now such that $\bar{A}_g(S)$ and $\underline{A}_g(S)$ are functions of the strike S and no longer depend on the extreme values of the spot price. Indeed, the CfD, by ensuring a fixed revenue stream, mitigates ambiguity. On the other hand, bounds $\bar{B}_g$ and $\underline{B}_g$ are unchanged.

The first-stage problem for producer g can then be rewritten as:

$$\inf_{\substack{K_g \geq 0 \\ W_g \in \mathbb{R}^{|\mathcal{C}|}}} C_g K_g + \sup_{\mathbb{Q} \in \mathcal{B}_{\varepsilon_g}(\hat{\mathbb{P}}_g)} \mathbb{E}_{\mathbb{Q}}\left[l_g\left(K_g, W_g, \xi_g(\boldsymbol{K}, \boldsymbol{W})\right)\right] \tag{40a}$$
$$\text{s.t.} \quad K_g \leq K_g^S \qquad [\mu_g^{cfd}], \tag{40b}$$

where the additional constraint limiting installed capacity derives from our assumption that producer g can not invest in greater capacity than what can be covered by the CfD, i.e. K_g^S.

Lemma 1 still applies to this problem and offers a linear reformulation, as in Section 2.2.1:

$$\inf_{\substack{K_g, x'_{igt}, \gamma^1_{igt}, \gamma^2_{igt}, \gamma^1_{igc}, \gamma^2_{igc} \geq 0 \\ \lambda_g, s_{ig}, W_{gc} \in \mathbb{R}}} C_g K_g + \lambda_g \varepsilon_g + \frac{1}{N}\sum_i s_{ig}$$

$$\begin{aligned}
\text{s.t.} \quad & K_g \leq K_g^S && [\mu_g^{cfd}] \\
& \forall i \in \mathcal{N}, t \in \mathcal{T}, \quad x'_{igt} \leq K_g && [\mu'_{igt}] \\
& \forall i \in \mathcal{N}, \quad \begin{cases} \sum_t (H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt})(c_{igt} - S) + \gamma^1_{igt}\bar{A}_g(S) - \gamma^2_{igt}\underline{A}_g(S) \\ + \sum_c (W_{gc} - \gamma^1_{igc} + \gamma^2_{igc})(p^1_c - p^2_{ic}) + \gamma^1_{igc}\bar{B}_g - \gamma^2_{igc}\underline{B}_g \end{cases} \leq s_{ig} && [\alpha_{ig}] \\
& \forall i \in \mathcal{N}, t \in \mathcal{T}, \quad -\lambda_g \leq H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt} \leq \lambda_g && [\beta^2_{igt}, \beta^1_{igt}] \\
& \forall c \in \mathcal{C}, \quad -\lambda_g \leq W_{gc} - \gamma^1_{igc} + \gamma^2_{igc} \leq \lambda_g && [\beta^2_{igc}, \beta^1_{igc}].
\end{aligned}$$

The equivalent KKT conditions of the problem for generator g, who has contracted a CfD at strike S for capacity K_g^S, are then derived as:

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \leq \mu'_{igt} \perp x'_{igt} - K_g \leq 0 \tag{41a}$$

$$\forall i \in \mathcal{N}, \quad 0 \geq \alpha_{ig} \perp \begin{cases} \sum_t (H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt})(c_{igt} - S) + \gamma^1_{igt}\bar{A}_g(S) - \gamma^2_{igt}\underline{A}_g(S) \\ + \sum_c (W_{gc} - \gamma^1_{igc} + \gamma^2_{igc})(p^1_c - p^2_{ic}) + \gamma^1_{igc}\bar{B}_g - \gamma^2_{igc}\underline{B}_g - s_{ig} \end{cases} \leq 0 \tag{41b}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \geq \beta^1_{igt} \perp H_t x'_{igt} - \gamma^1_{igt} + \gamma^2_{igt} - \lambda_g \leq 0 \tag{41c}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \geq \beta^2_{igt} \perp -H_t x'_{igt} + \gamma^1_{igt} - \gamma^2_{igt} - \lambda_g \leq 0 \tag{41d}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \geq \beta^1_{igc} \perp W_{gc} - \gamma^1_{igc} + \gamma^2_{igc} - \lambda_g \leq 0 \tag{41e}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \geq \beta^2_{igc} \perp -W_{gc} + \gamma^1_{igc} - \gamma^2_{igc} - \lambda_g \leq 0 \tag{41f}$$

$$0 \geq \mu_g^{cfd} \perp K_g - K_g^S \leq 0 \tag{41g}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \leq x'_{igt} \perp -\mu'_{igt} - H_t\beta^2_{igt} + H_t\beta^1_{igt} + \alpha_{ig} H_t (c_{igt} - S) \leq 0 \tag{41h}$$

$$0 \leq K_g \perp -C_g + \sum_i \sum_t \mu'_{igt} + \mu_g^{cfd} \leq 0 \tag{41i}$$

$$\forall c \in \mathcal{C}, \quad W_{gc} \perp \sum_i \alpha_{ig}(p^1_c - p^2_{ic}) + \beta^1_{igc} - \beta^2_{igc} = 0 \tag{41j}$$

$$\lambda_g \perp -\varepsilon_g - \sum_t \sum_i (\beta^1_{igt} + \beta^2_{igt}) - \sum_c \sum_i (\beta^1_{igc} + \beta^2_{igc}) = 0 \tag{41k}$$

$$\forall i \in \mathcal{N}, \quad s_{ig} \perp -\frac{1}{N} - \alpha_{ig} = 0 \tag{41l}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \leq \gamma^1_{igt} \perp \beta^2_{igt} - \beta^1_{igt} - \alpha_{ig}(c_{igt} - S) + \alpha_{ig}\bar{A}_g(S) \leq 0 \tag{41m}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad 0 \leq \gamma^2_{igt} \perp -\beta^2_{igt} + \beta^1_{igt} + \alpha_{ig}(c_{igt} - S) - \alpha_{ig}\underline{A}_g(S) \leq 0 \tag{41n}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \leq \gamma^1_{igc} \perp \beta^2_{igc} - \beta^1_{igc} - \alpha_{ig}(p^1_c - p^2_{ic}) + \alpha_{ig}\bar{B}_g \leq 0 \tag{41o}$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad 0 \leq \gamma^2_{igc} \perp -\beta^2_{igc} + \beta^1_{igc} + \alpha_{ig}(p^1_c - p^2_{ic}) - \alpha_{ig}\underline{B}_g \leq 0. \tag{41p}$$

The representative consumer exhibits the same behavior as in Section 2.2.2, except that it now accounts, in a distributionally robust way, for the settlement payment of the CfD it exchanges with producer g. In realization i, the second-stage problem for the consumer is reformulated accordingly as

$$l_d(W_d, \xi_{id}) = \min_{e_{it} \geq 0} \sum_t H_t(e_{it} - D_{it})(PC - p_{it}) + \sum_c W_{dc}(p_c^1 - p_{ic}^2) + \sum_t H_t x_{igt}(S - p_{it}), \tag{42}$$

where the last term reflects the financial transaction with producer g according to the CfD agreement. Note that the consumer controls neither generator g's production nor the spot price p_{it}, so we regard this last term of the objective as an *additional ambiguous* parameter.

The entire vector of ambiguous parameters for the consumer is then (here again, through a slight abuse of the notation, we still denote this vector by ξ_d)

$$\forall i \in \mathcal{N}, \xi_{id} = \begin{pmatrix} PC - p_{it} \\ S - p_{it} \\ p_c^1 - p_{ic}^2 \end{pmatrix} \in \mathbb{R}^{2T+|\mathcal{C}|}.$$

We now denote by $\underline{A}_d(S), \bar{A}_d(S)$ the bounds of the two first uncertain parameters of ξ_{id}. We do not change the support for the last parameter $p_c^1 - p_{ic}^2$.

The first-stage problem for the consumer can be written as in Section 2.2.2:

$$\inf_{W_d \in \mathbb{R}^{|\mathcal{C}|}} \sup_{\mathbb{Q} \in \mathcal{B}_{\varepsilon_d}(\hat{\mathbb{P}}_d)} \mathbb{E}_{\mathbb{Q}} \left[l_d\Big(W_d, \xi_d(\boldsymbol{K}, \boldsymbol{W})\Big) \right]. \tag{43}$$

According to Lemma 1, this problem reduces again to the following linear program:

$$\inf_{\substack{W_{dc}, \lambda_d, s_{id} \in \mathbb{R} \\ e'_{it}, \gamma^1_{idt}, \gamma^2_{idt}, \gamma^1_{idc}, \gamma^2_{idc} \geq 0}} \quad \lambda_d \varepsilon_d + \frac{1}{N} \sum_i s_{id} \tag{44}$$

$$\forall i \in \mathcal{N}, \quad \begin{cases} \sum_t [H_t(e'_{it} - D_{it}) - \gamma^1_{idt} + \gamma^2_{idt}](PC - p_{it}) + \gamma^1_{idt}\bar{A}_d(S) - \gamma^2_{idt}\underline{A}_d(S) \\ + \sum_t [H_t x'_{igt} - \gamma^1_{idt} + \gamma^2_{idt}](S - p_{it}) + \gamma^1_{idt}\bar{A}_d(S) - \gamma^2_{idt}\underline{A}_d(S) \\ + \sum_c [W_{dc} - \gamma^1_{idc} + \gamma^2_{idc}](p_c^1 - p_{ic}^2) + \gamma^1_{idc}\bar{B}_d - \gamma^2_{idc}\underline{B}_d \end{cases} \leq s_{id} \qquad [\alpha_{id}]$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad -\lambda_d \leq H_t(e'_{it} - D_{it}) - \gamma^1_{idt} + \gamma^2_{idt} \leq \lambda_d \qquad [\beta^2_{idt}, \beta^1_{idt}]$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \quad -\lambda_d \leq H_t x'_{igt} - \gamma^1_{idt} + \gamma^2_{idt} \leq \lambda_d \qquad [\beta^2_{idt}, \beta^1_{idt}]$$

$$\forall i \in \mathcal{N}, c \in \mathcal{C}, \quad -\lambda_d \leq W_{dc} - \gamma^1_{idc} + \gamma^2_{idc} \leq \lambda_d \qquad [\beta^2_{idc}, \beta^1_{idc}],$$

This linear program is equivalent to the KKT conditions specified in Section 2.2.2, except for a modification of (20a):

$$\forall i \in \mathcal{N}, \qquad 0 \geq \alpha_{id} \perp \begin{cases} \sum_t [H_t(e'_{it} - D_{it}) - \gamma^1_{idt} + \gamma^2_{idt}](PC - p_{it}) + \gamma^1_{idt}\bar{A}_d(S) - \gamma^2_{idt}\underline{A}_d(S) \\ + \sum_t [H_t x'_{igt} - \gamma^1_{idt} + \gamma^2_{idt}](S - p_{it}) + \gamma^1_{idt}\bar{A}_d(S) - \gamma^2_{idt}\underline{A}_d(S) \\ + \sum_c [W_{dc} - \gamma^1_{idc} + \gamma^2_{idc}](p^1_c - p^2_{ic}) + \gamma^1_{idc}\bar{B}_d(S) - \gamma^2_{idc}\underline{B}_d(S) \end{cases} - s_{id} \leq 0, \tag{45}$$

and for the following new complementarity conditions associated with the consumer's having to be distributionally robust with respect to the uncertain CfD payments:[7]

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \qquad 0 \geq \beta^1_{idt} \perp H_t x'_{igt} - \gamma^1_{idt} + \gamma^2_{idt} - \lambda_d \leq 0 \tag{46a}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \qquad 0 \geq \beta^2_{idt} \perp -H_t x'_{igt} + \gamma^1_{idt} - \gamma^2_{idt} - \lambda_d \leq 0 \tag{46b}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \qquad 0 \leq \gamma^1_{idt} \perp \beta^2_{idt} - \beta^1_{idt} - \alpha_{id}(S - p_{it}) + \alpha_{id}\bar{A}_d(S) \leq 0 \tag{46c}$$

$$\forall i \in \mathcal{N}, t \in \mathcal{T}, \qquad 0 \leq \gamma^2_{idt} \perp -\beta^2_{idt} + \beta^1_{idt} + \alpha_{id}(S - p_{it}) - \alpha_{id}\underline{A}_d(S) \leq 0. \tag{46d}$$

D An equilibrium formulation of the economy under risk aversion

We now provide a formulation of our problem when agents are averse to risk but not necessarily to ambiguity. The realizations of the random variables (demand, fuel costs, and prices) constitute the set of scenarios, which we still denote by $i \in \{1, 2, ..., N\}$ and to which we assign equal probability $\frac{1}{N}$. The rest of the notation remains unchanged; in particular, we still keep the classical two-stage formalism. As is now standard in the literature, we resort to risk measures to model market agents' aversion to risk (Artzner et al. [1999] and Shapiro et al. [2021]). Because the financial market is never complete, every agent has to value its stochastic profit via its own risk measure. For ease of exposition, we formulate our problem with the CVaR (Conditional Value at Risk), which can be expressed linearly as shown in Rockafellar and Uryasev [2000]. We denote by ε_g^{CVaR} producers' levels of risk aversion and by ε_d^{CVaR} that of the consumer. The obtained model is of the complementarity form, which we now present and refer to Ferris and Philpott [2022], for instance, for all derivation details.

[7] We draw the reader's attention to the fact that the decisions the representative consumer takes regarding foreseen load curtailment (e') and eventual investment in contracts (W_{dc}) are unaffected by this payment.

The KKT conditions for producers are

$$0 \geq \mu_{igt} \perp x_{igt} - K_g \leq 0 \tag{47a}$$

$$0 \leq x_{igt} \perp p_{it} - c_{igt} + \mu_{igt} \leq 0 \tag{47b}$$

$$0 \leq y^*_{ig} \perp \eta^*_g + \gamma^*_{ig} - \sum_c W_{gc} p^2_{ic} + \sum_t H_t \mu_{igt} K_g \leq 0 \tag{47c}$$

$$0 \leq \gamma^*_{ig} \perp y^*_{ig} - \frac{1}{(1-\varepsilon^{CVaR}_g)N} \leq 0 \tag{47d}$$

$$\eta^*_g \perp \sum_i y^*_{ig} - 1 = 0 \tag{47e}$$

$$0 \leq K_g \perp -C_g - \sum_i y^*_{ig} \sum_t H_t \mu_{igt} \leq 0 \tag{47f}$$

$$W_{gc} \perp p^1_c - \sum_i y^*_{ig} p^2_{ic} = 0, \tag{47g}$$

where y^*_{ig}, $i \in \{1, 2, ..., N\}$ is the risk-adjusted probability associated with producer g. The risk-adjusted profit agent g earns, RAP_g, is:

$$RAP_g = K_g. \left(\sum_i y^*_{ig} \sum_t H_t(-\mu_{igt}) - C_g \right) + \sum_c W_{gc} \left(\sum_i y^*_{ig} p^2_{ic} - p^1_c \right).$$

The KKT conditions of the consumer are

$$0 \leq y^*_{id} \perp \eta^*_d + \gamma^*_{id} - \sum_c p^2_{ic} W_{dc} - \sum_t H_t(PC - p_{it})(D_{it} - e_{it}) \leq 0 \tag{48a}$$

$$0 \leq \gamma^*_{id} \perp y^*_{id} - \frac{1}{(1-\varepsilon^{CVaR}_d)N} \leq 0 \tag{48b}$$

$$\eta^* \perp \sum_i y^*_{id} - 1 = 0 \tag{48c}$$

$$W_{dc} \perp p^1_c - \sum_i y^*_{id} p^2_{ic} = 0 \tag{48d}$$

$$0 \leq e_{it} \perp p_{it} - PC \leq 0, \tag{48e}$$

where y^*_{id}, $i \in \{1, 2, ..., N\}$ is the risk-adjusted probability of the consumer. The risk-adjusted consumer surplus, $RACS$, is

$$RACS = \sum_i y^*_{id} \sum_t H_t(PC - p_{it})(D_{it} - e_{it}) + \sum_c W_{dc} \left(\sum_i y^*_{id} p^2_{ic} - p^1_c \right).$$

Finally, we define risk-adjusted welfare RAW as follows:

$$RAW = \sum_{g \in \mathcal{G}} RAP_g + RACS. \tag{49}$$

References

I. Abada and A. Ehrenmann. When Market Incompleteness Is Preferable to Market Power: Insights from Power Markets. SSRN Electronic Journal, 2023.

I. Abada, G. de Maere d Aertrycke, and Y. Smeers. On the multiplicity of solutions in generation capacity investment models with incomplete markets: A risk-averse stochastic equilibrium approach. Mathematical Programming, 165:5–69, 9 2017a.

I. Abada, A. Ehrenmann, and Y. Smeers. Modeling gas markets with endogenous long-term contracts. Operations Research, 65:856–877, 8 2017b.

A. Arrigo, C. Ordoudis, J. Kazempour, Z. De Grève, J.-F. Toubeau, and F. Vallée. Wasserstein distributionally robust chance-constrained optimization for energy and reserve dispatch: An exact and physically-bounded formulation. European Journal of Operational Research, 296 (1):304–322, 2022.

P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath. Coherent measures of risk. Mathematical Finance, 9:203–228, 7 1999.

D. Bertsimas, M. Sim, and M. Zhang. Adaptive distributionally robust optimization. Management Science, 65(2):604–618, 2019.

M. Boiteux. Peak-load pricing. The Journal of Business, 33(2):157–179, 1960.

S. Boyd and L. Vandenberghe. Convex optimization. 7th printing. Cambridge University Press, 2004.

J. H. Cochrane. Asset pricing: Revised edition. Princeton university press, 2009.

CRE. Rapport de surveillance des marchés de gros de l'électricité et du gaz naturel 2023. Technical report, Commission de Régulation de l'Energie, 2024.

G. de Maere d Aertrycke and Y. Smeers. Liquidity risks on power exchanges: A generalized Nash equilibrium model. Mathematical Programming, 140:381–414, 9 2013.

G. de Maere d Aertrycke, A. Ehrenmann, and Y. Smeers. Investment with incomplete markets for risk: The need for long-term contracts. Energy Policy, 105:571–583, 6 2017.

G. de Maere d'Aertrycke and Y. Smeers. Liquidity risks on power exchanges: a generalized nash equilibrium model. Mathematical Programming, 2013.

G. Debreu. A social equilibrium existence theorem. Proceedings of the National Academy of Sciences, 38(10):886–893, 1952.

E. Dimanchev, S. A. Gabriel, L. Reichenberg, and M. Korpås. Consequences of the missing risk market problem for power system emissions. Energy Economics, 136:107639, 2024.

A. Downward, D. Young, and G. Zakeri. Electricity contracting and policy choices under risk-aversion. Operations Research, 143, 2012.

A. Downward, D. Young, and G. Zakeri. Electricity retail contracting under risk-aversion. European Journal of Operational Research, 251(3):846–859, 2016.

R. Egging and F. Holz. Risks in global natural gas markets: Investment, hedging and trade. Energy Policy, 94:468–479, 2016.

A. Ehrenmann and Y. Smeers. Stochastic Equilibrium Models for Generation Capacity Expansion. Springer New York, 2011.

A. Esteban-Pérez and J. M. Morales. Distributionally robust optimal power flow with contextual information. European Journal of Operational Research, 306(3):1047–1058, 2023.

M. Ferris and A. Philpott. Dynamic risked equilibrium. Operations Research, 70(3):1933–1952, 2022.

D. Finon. Investment risk allocation in decentralised electricity markets. the need of long-term contracts and vertical integration. OPEC Energy Review, 32:150–183, 6 2008.

H. Föllmer and A. Schied. Convex measures of risk and trading constraints. Finance and Stochastics, 6:429–447, 10 2002.

J. Gao, Y. Xu, J. Barreiro-Gomez, M. Ndong, M. Smyrnakis, and H. Tembine. Distributionally robust optimization. In Optimization Algorithms: Examples. Intechopen London, UK, 2018.

I. Gilboa and D. Schmeidler. Maxmin expected utility with non-unique prior. Journal of mathematical economics, 18(2):141–153, 1989.

J. Goh and M. Sim. Distributionally robust optimization and its tractable approximations. Operations research, 58(4-part-1):902–917, 2010.

IEA. Net zero roadmap: A global pathway to keep the 1.5 °c goal in reach. Technical report, IEA, 2023.

P. L. Joskow. Competitive electricity markets and investment in new generating capacity. AEI-Brookings Joint Center Working Paper No. 06-14, 2006.

P. L. Joskow. Capacity payments in imperfect electricity markets: Need and design. Utilities policy, 16(3):159–170, 2008.

P. Klibanoff, M. Marinacci, and S. Mukerji. A smooth model of decision making under ambiguity. Econometrica, 73(6):1849–1892, 2005.

P. Klibanoff, S. Mukerji, K. Seo, and L. Stanca. Foundations of ambiguity models under symmetry: α-meu and smooth ambiguity. Journal of Economic Theory, 199:105202, 2022.

J. Mays and J. Jenkins. Electricity markets under deep decarbonization. USAEE Working Paper No. 22-550, 2022.

P. Mohajerin Esfahani and D. Kuhn. Data-driven distributionally robust optimization using the Wasserstein metric: performance guarantees and tractable reformulations. Mathematical Programming, 171(1-2):115–166, 2018.

D. Newbery. Missing money and missing markets: Reliability, capacity auctions and interconnectors. Energy Policy, 94:401–410, 7 2016.

A. Philpott, M. Ferris, and R. Wets. Equilibrium, uncertainty and risk in hydro-thermal electricity systems. Mathematical Programming, 157:483–513, 6 2016.

R. C. Pietzcker, S. Osorio, and R. Rodrigues. Tightening EU ETS targets in line with the European Green Deal: Impacts on the decarbonization of the EU power sector. Applied Energy, 293: 116914, 2021.

F. Pourahmadi and J. Kazempour. Distributionally robust generation expansion planning with unimodality and risk constraints. IEEE Transactions on Power Systems, 36(5):4281–4295, 2021.

H. Rahimian and S. Mehrotra. Distributionally robust optimization: A review. arXiv preprint arXiv:1908.05659, 2019.

D. Ralph and Y. Smeers. Pricing risk under risk measures: An introduction to stochastic-endogenous equilibria. Technical report, Cambridge Judge Business School, University of Cambridge, 2011.

D. Ralph and Y. Smeers. Risk trading and endogenous probabilities in investment equilibria. SIAM Journal on Optimization, 25:2589–2611, 1 2015.

R. T. Rockafellar and S. Uryasev. Optimization of conditional value-at-risk. The Journal of Risk, 2(3):21–41, 2000.

A. Shapiro, D. Dentcheva, and A. Ruszczynski. Lectures on stochastic programming: Modeling and theory. Society for Industrial and Applied Mathematics., 2021.

H. Sun and H. Xu. Convergence analysis for distributionally robust optimization and equilibrium problems. Mathematics of Operations Research, 41(2):377–401, 2016.

B. P. Van Parys, P. M. Esfahani, and D. Kuhn. From data to decisions: Distributionally robust optimization is optimal. Management Science, 67(6):3387–3402, 2021.

W. Wiesemann, D. Kuhn, and M. Sim. Distributionally robust convex optimization. Operations research, 62(6):1358–1376, 2014.